To the unaided eye, the night sky is spangled with several thousand stars, each appearing as a bright pinpoint of light. With a pair of binoculars, you can see some 10,000 other, fainter stars; with a 15-cm (6-in.) telescope, the total rises to more than 2 million. Astronomers now know that there are in excess of 100 billion \((10^{11})\) stars in our Milky Way Galaxy alone.

But what are these distant pinpoints? To the great thinkers of ancient Greece, the stars were bits of light embedded in a vast sphere with the Earth at its center. They thought the stars were composed of a mysterious “fifth element,” quite unlike anything found on Earth.

Today, we know that the stars are made of the same chemical elements found on Earth. We know their sizes, their temperatures, their masses, and something of their internal structures. We understand, too, why the stars in the accompanying image come in a range of beautiful colors: Blue stars have high surface temperatures, while the surface temperatures of red and yellow stars are relatively low.

How have we learned these things? How can we know the nature of the stars, objects so distant that their light takes years or centuries to reach us? In this chapter, we will learn about the measurements and calculations that astronomers make to determine the properties of stars. We will also take a first look at the Hertzsprung-Russell diagram, an important tool that helps astronomers systematize the wealth of available information about the stars. In later chapters, we will use this diagram to help us understand how stars are born, evolve, and eventually die.

### Learning Goals

By reading the sections of this chapter, you will learn:

- **17-1** How we can measure the distances to the stars
- **17-2** How we measure a star’s brightness and luminosity
- **17-3** The magnitude scale for brightness and luminosity
- **17-4** How a star’s color indicates its temperature
- **17-5** How a star’s spectrum reveals its chemical composition
- **17-6** How we can determine the sizes of stars
- **17-7** How H-R diagrams summarize our knowledge of the stars
- **17-8** How we can deduce a star’s size from its spectrum
- **17-9** How we can use binary stars to measure the masses of stars
- **17-10** How we can learn about binary stars in very close orbits
- **17-11** What eclipsing binaries are and what they tell us about the sizes of stars

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Some stars in this cluster (called M39) are distinctly blue in color, while others are yellow or red. (Heidi Schweiker/NOAO/AURA/NSF)
huge distances, it must be that the luminosity of the stars—that is, how much energy they emit into space per second—is comparable to or greater than that of the Sun. Just as for the Sun, the only explanation for such tremendous luminosities is that thermonuclear reactions are occurring within the stars (see Section 16-1).

Clearly, then, it is important to know how distant the stars are. But how do we measure these distances? You might think this is done by comparing how bright different stars appear. Perhaps the star Betelgeuse in the constellation Orion appears bright because it is relatively close, while the dimmer and less conspicuous star Polaris (the North Star, in the constellation Ursa Minor) is farther away.

But this line of reasoning is incorrect: Polaris is actually closer to us than Betelgeuse! How bright a star appears is not a good indicator of its distance. If you see a light on a darkened road, it could be a motorcycle headlight a kilometer away or a person holding a flashlight just a few meters away. In the same way, a bright star might be extremely far away but have an unusually high luminosity, and a dim star might be relatively close but have a rather low luminosity. Astronomers must use other techniques to determine the distances to the stars.

**Parallax and the Distances to the Stars**

The most straightforward way of measuring stellar distances uses an effect called **parallax**. This is the apparent displacement of an object because of a change in the observer’s point of view (Figure 17-1). To see how parallax works, hold your arm out straight in front of you. Now look at the hand on your outstretched arm, first with your left eye closed, then with your right eye closed. When you close one eye and open the other, your hand appears to shift back and forth against the background of more distant objects.

The closer the object you are viewing, the greater the parallax shift. To see this, repeat the experiment with your hand held closer to your face. Your brain analyzes such parallax shifts constantly as it compares the images from your left and right eyes, and in this way determines the distances to objects around you. This is the origin of depth perception.

To measure the distance to a star, astronomers measure the parallax shift of the star using two points of view that are as far apart as possible—at opposite sides of the Earth’s orbit. The direction from Earth to a nearby star changes as our planet orbits the Sun, and the nearby star appears to move back and forth against the background of more distant stars (Figure 17-2). This motion is called **stellar parallax**. The parallax \( p \) of a star is equal to half the angle through which the star’s apparent position shifts as the Earth moves from one side of its orbit to the other. The larger the parallax \( p \), the smaller the distance \( d \) to the star (compare Figure 17-2a with Figure 17-2b).

It is convenient to measure the distance \( d \) in **parsecs**. A star with a parallax angle of 1 second of arc \( (p = 1 \text{ arcsec}) \) is at a distance of 1 parsec \( (d = 1 \text{ pc}) \). (The word “parsec” is a contraction of the phrase “the distance at which a star has a parallax of one arcsecond.” Recall from Section 1-7 that 1 parsec equals 3.26 light-years, \( 3.09 \times 10^{13} \text{ km} \), or \( 206,265 \text{ AU} \); see Figure 1-12.) If the angle \( p \) is measured in arcseconds, then the distance \( d \) to the star in parsecs is given by the following equation:

**Relation between a star’s distance and its parallax**

\[
 d = \frac{1}{p}
\]

\( d = \) distance to a star, in parsecs  
\( p = \) parallax angle of that star, in arcseconds

This simple relationship between parallax and distance in parsecs is one of the main reasons that astronomers usually measure cosmic distances in parsecs rather than light-years. For example, a star whose parallax is \( p = 0.1 \text{ arcsec} \) is at a distance \( d = \frac{1}{0.1} = 10 \text{ parsecs} \) from the Earth. Barnard’s star, named for the American astronomer Edward E. Barnard, has a parallax of 0.547 arcsec. Hence, the distance to this star is:

\[
 d = \frac{1}{p} = \frac{1}{0.545} = 1.83 \text{ pc}
\]

Because 1 parsec is 3.26 light-years, this can also be expressed as

\[
 d = 1.83 \text{ pc} \times \frac{3.26 \text{ ly}}{1 \text{ pc}} = 5.98 \text{ ly}
\]

All known stars have parallax angles less than one arcsecond. In other words, the closest star is more than 1 parsec away. Such
small parallax angles are difficult to detect, so it was not until 1838 that the first successful parallax measurements were made by the German astronomer and mathematician Friedrich Wilhelm Bessel. He found the parallax angle of the star 61 Cygni to be just 1/3 arcsec—equal to the angular diameter of a dime at a distance of 11 kilometers, or 7 miles. He thus determined that this star is about 3 pc from the Earth. (Modern measurements give a slightly smaller parallax angle, which means that 61 Cygni is actually more than 3 pc away.) The star Proxima Centauri has the largest known parallax angle, 0.772 arcsec, and hence is the closest known star (other than the Sun); its distance is $1/(0.772) = 1.30$ pc.

Appendix 4 at the back of this book lists all the stars within 4 pc of the Sun, as determined by parallax measurements. Most of these stars are far too dim to be seen with the naked eye, which is why their names are probably unfamiliar to you. By contrast, the majority of the familiar, bright stars in the nighttime sky (listed in Appendix 5) are so far away that their parallaxes cannot be measured from the Earth’s surface. They appear bright not because they are close, but because they are far more luminous than the Sun. The brightest stars in the sky are not necessarily the nearest stars!

**Measuring Parallax from Space**

Parallax angles smaller than about 0.01 arcsec are extremely difficult to measure from the Earth, in part because of the blurring effects of the atmosphere. Therefore, the parallax method used with ground-based telescopes can give fairly reliable distances only for stars nearer than about $1/0.01 = 100$ pc. But an observatory in space is unhampered by the atmosphere. Observations made from spacecraft therefore permit astronomers to measure even smaller parallax angles and thus determine the distances to more remote stars.

In 1989 the European Space Agency (ESA) launched the satellite *Hipparcos*, an acronym for *Hi*gh *P*recision *Par*allax *Col*lecting *S*atellite (and a commemoration of the ancient Greek astronomer Hipparchus, who created one of the first star charts). Over more than three years of observations, the telescope aboard *Hipparcos* was used to measure the parallaxes of 118,000 stars with an accuracy of 0.001 arcsecond. This has enabled astronomers to determine stellar distances out to several hundred parsecs, and with much greater precision than has been possible with ground-based observations. In the years to come, astronomers will increasingly turn to space-based observations to determine stellar distances.

Unfortunately, most of the stars in the Galaxy are so far away that their parallax angles are too small to measure even with an orbiting telescope. Later in this chapter, we will discuss a technique that can be used to find the distances to these more remote stars. In Chapters 24 and 26 we will learn about other techniques that astronomers use to determine the much larger distances to galaxies beyond the Milky Way. These techniques also
Stellar Motions

Stars can move through space in any direction. The space velocity of a star describes how fast and in what direction it is moving. As the accompanying figure shows, a star’s space velocity \( v \) can be broken into components parallel and perpendicular to our line of sight.

The component perpendicular to our line of sight—that is, across the plane of the sky—is called the star’s tangential velocity \( (v_t) \). To determine it, astronomers must know the distance to a star \( (d) \) and its proper motion \( (\mu, \) the Greek letter \( \mu \)), which is the number of arcseconds that the star appears to move per year on the celestial sphere. Proper motion does not repeat itself yearly, so it can be distinguished from the apparent back-and-forth motion due to parallax. In terms of a star’s distance and proper motion, its tangential velocity (in km/s) is

\[
v_t = 4.74 \mu d
\]

where \( \mu \) is in arcseconds per year and \( d \) is in parsecs. For example, Barnard’s star (Figure 17-3) has a proper motion of 10.358 arcseconds per year and a distance of 1.83 pc. Hence, its tangential velocity is

\[
v_t = 4.74(10.358)(1.83) = 89.8 \text{ km/s}
\]

The component of a star’s motion parallel to our line of sight—that is, directly toward us or directly away from us—is its radial velocity \( (v_r) \). It can be determined from measurements of the Doppler shifts of the star’s spectral lines (see Section 5-9 and Box 5-6). If a star is approaching us, the wavelengths of all of its spectral lines are decreased (blueshifted); if the star is receding from us, the wavelengths are increased (redshifted). The radial velocity \( v_r \) is related to the wavelength shift by the equation

\[
\lambda = \frac{\lambda_0}{\lambda_0} = \frac{v_r}{c}
\]

In this equation, \( \lambda \) is the wavelength of light coming from the star, \( \lambda_0 \) is what the wavelength would be if the star were not moving, and \( c \) is the speed of light. As an illustration, a particular spectral line of iron in the spectrum of Barnard’s star has a wavelength \( (\lambda) \) of 516.445 nm. As measured in a laboratory on the Earth, the same spectral line has a wavelength \( (\lambda_0) \) of 516.629 nm. Thus, for Barnard’s star, our equation becomes

\[
\frac{516.445 \text{ nm} - 516.629 \text{ nm}}{516.629 \text{ nm}} = -0.000356 = \frac{v_r}{c}
\]

Solving this equation for the radial velocity \( v_r \), we find

\[
v_r = (-0.000356) c = (-0.000356)(3.00 \times 10^5 \text{ km/s}) = -107 \text{ km/s}
\]

The negative sign means that Barnard’s star is moving toward us. You can check this interpretation by noting that the wavelength \( \lambda = 516.445 \text{ nm} \) received from Barnard’s star is less than the laboratory wavelength \( \lambda_0 = 516.629 \text{ nm} \); hence, the light from the star is blueshifted, which indeed means that the star is approaching. If the star were receding, its radial velocity would be positive.

The illustration shows that the tangential velocity and radial velocity form two sides of a right triangle. The long side (hypotenuse) of this triangle is the space velocity \( (v) \). From the Pythagorean theorem, the space velocity is

\[
v = \sqrt{v_t^2 + v_r^2}
\]

For Barnard’s star, the space velocity is

\[
v = \sqrt{(89.4 \text{ km/s})^2 + (-107 \text{ km/s})^2} = 140 \text{ km/s}
\]

Therefore, Barnard’s star is moving through space at a speed of 140 km/s (503,000 km/h, or 312,000 mi/h) relative to the Sun.

Determining the space velocities of stars is essential for understanding the structure of the Galaxy. Studies show that the stars in our local neighborhood are moving in wide orbits around the center of the Galaxy, which lies some 8000 pc (26,000 light-years) away in the direction of the constellation Sagittarius (the Archer). While many of the orbits are roughly circular and lie in nearly the same plane, others are highly elliptical or steeply inclined to the galactic plane. We will see in Chapter 23 how the orbits of stars and gas clouds reveal the Galaxy’s spiral structure.
help us understand the overall size, age, and structure of the universe.

The Importance of Parallax Measurements

Because it can be used only on relatively close stars, stellar parallax might seem to be of limited usefulness. But parallax measurements are the cornerstone for all other methods of finding the distances to remote objects. These other methods require a precise and accurate knowledge of the distances to nearby stars, as determined by stellar parallax. Hence, any inaccuracies in the parallax angles for nearby stars can translate into substantial errors in measurement for the whole universe. For this reason, astronomers are continually trying to perfect their parallax-measuring techniques.

Stellar parallax is an apparent motion of stars caused by the Earth’s orbital motion around the Sun. But stars are not fixed objects and actually do move through space. As a result, stars change their positions on the celestial sphere (Figure 17-3), and they move either toward or away from the Sun. These motions are sufficiently slow, however, that changes in the positions of the stars are hardly noticeable over a human lifetime. Box 17-1 describes how astronomers study these motions and what insights they gain from these studies.

If a star’s distance is known, its luminosity can be determined from its apparent brightness

All the stars you can see in the nighttime sky shine by thermonuclear fusion, just as the Sun does (see Section 16-1). But they are by no means merely identical copies of the Sun. Stars differ in their luminosity \( L \), the amount of light energy they emit each second. Luminosity is usually measured either in watts (1 watt, or 1 W, is 1 joule per second) or as a multiple of the Sun’s luminosity \( L_\odot \), equal to \( 3.86 \times 10^{26} \) W. Most stars are less luminous than the Sun, but some blaze forth with a million times the Sun’s luminosity. Knowing a star’s luminosity is essential for determining the star’s history, its present-day internal structure, and its future evolution.

Luminosity, Apparent Brightness, and the Inverse-Square Law

To determine the luminosity of a star, we first note that as light energy moves away from its source, it spreads out over increasingly larger regions of space. Imagine a sphere of radius \( d \) centered on the light source, as in Figure 17-4. The amount of energy that passes each second through a square meter of the sphere’s surface area is the total luminosity of the source \( L \) divided by the total surface area of the sphere (equal to \( 4\pi d^2 \)). This quantity is called the apparent brightness of the light, or just brightness \( b \), because how bright a light source appears depends on how much light energy per second enters through the area of a light detector (such as your eye). Apparent brightness is measured in watts per square meter (W/m²). Written in the form of an equation, the relationship between apparent brightness and luminosity is

\[
b = \frac{L}{4\pi d^2}
\]

With greater distance from the star, its light is spread over a larger area and its apparent brightness is less.

Apparent brightness is a measure of how faint a star looks to us; luminosity is a measure of the star’s total light output.

The Nature of the Stars

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**BOX 17-2** 

**Luminosity, Distance, and Apparent Brightness**

The inverse-square law (Section 17-2) relates a star’s luminosity, distance, and apparent brightness to the corresponding quantities for the Sun:

\[
\frac{L}{L_{\odot}} = \left( \frac{d}{d_{\odot}} \right)^2 \frac{b}{b_{\odot}}
\]

We can use a similar equation to relate the luminosities, distances, and apparent brightnesses of any two stars, which we call star 1 and star 2:

\[
\frac{L_1}{L_2} = \left( \frac{d_1}{d_2} \right)^2 \frac{b_1}{b_2}
\]

**EXAMPLE:** The star \( \epsilon \) (epsilon) Eridani is 3.23 pc from Earth. As seen from Earth, this star appears only \( 6.73 \times 10^{-13} \) as bright as the Sun. What is the luminosity of \( \epsilon \) Eridani compared with that of the Sun?

**Situation:** We are given the distance to \( \epsilon \) Eridani \((d = 3.23 \text{ pc}) \) and this star’s brightness compared to that of the Sun \((b/b_{\odot} = 6.73 \times 10^{-13}) \). Our goal is to find the ratio of the luminosity of \( \epsilon \) Eridani to that of the Sun, that is, the quantity \( L/L_{\odot} \).

**Tools:** Since we are asked to compare this star to the Sun, we use the first of the two equations given above, \( L/L_{\odot} = (d/d_{\odot})^2(b/b_{\odot}) \), to solve for \( L/L_{\odot} \).

**Answer:** Our equation requires the ratio of the star’s distance to the Sun’s distance, \( d/d_{\odot} \). The distance from the Earth to the Sun is \( d_{\odot} = 1 \text{ AU} \). To calculate the ratio \( d/d_{\odot} \), we must express both distances in the same units. There are 206,265 AU in 1 parsec, so we can write the distance to \( \epsilon \) Eridani as \( d = (3.23 \text{ pc})(206,265 \text{ AU/pc}) = 6.66 \times 10^5 \text{ AU} \). Hence, the ratio of distances is \( d/d_{\odot} = (6.66 \times 10^5 \text{ AU})/(1 \text{ AU}) = 6.66 \times 10^5 \). Then we find that the ratio of the luminosity of \( \epsilon \) Eridani \((L) \) to the Sun’s luminosity \((L_{\odot}) \) is

\[
\frac{L}{L_{\odot}} = \left( \frac{d}{d_{\odot}} \right)^2 \frac{b}{b_{\odot}} = (6.66 \times 10^5)^2 \times (6.73 \times 10^{-13}) = 0.30
\]

**Review:** This result means that \( \epsilon \) Eridani is only 0.30 as luminous as the Sun; that is, its power output is only 30% as great.

**EXAMPLE:** Suppose star 1 is at half the distance of star 2 (that is, \( d_1/d_2 = 1/2 \)) and that star 1 appears twice as bright as star 2 (that is, \( b_1/b_2 = 2 \)). How do the luminosities of these two stars compare?

**Situation:** For these two stars, we are given the ratio of distances \((d_1/d_2) \) and the ratio of apparent brightnesses \((b_1/b_2) \). Our goal is to find the ratio of their luminosities \((L_1/L_2) \).

**Tools:** Since we are comparing two stars, neither of which is the Sun, we use the second of the two equations above:

\[
\frac{L_1}{L_2} = (d_1/d_2)^2(b_1/b_2)
\]

**Answer:** Plugging in to our equation, we find

\[
\frac{L_1}{L_2} = \left( \frac{d_1}{d_2} \right)^2 \frac{b_1}{b_2} = \left( \frac{1}{2} \right)^2 \times 2 = \frac{1}{2}
\]

**Review:** This result says that star 1 has only one-half the luminosity of star 2. Despite this, star 1 appears brighter than star 2 because it is closer to us.

The two equations above are also useful in the method of spectroscopic parallax, which we discuss in Section 17-8. It turns out that a star’s luminosity can be determined simply by analyzing the star’s spectrum. If the star’s apparent brightness is also known, the star’s distance can be calculated. The inverse-square law can be rewritten as an expression for the ratio of the star’s distance from the Earth \((d) \) to the Earth-Sun distance \((d_{\odot}) \):

\[
\frac{d}{d_{\odot}} = \sqrt{\frac{L/L_{\odot}}{(b/b_{\odot})}}
\]

**Inverse-square law relating apparent brightness and luminosity**

\[
b = \frac{L}{4\pi d^2}
\]

- \( b \) = apparent brightness of a star’s light, in \( \text{W/m}^2 \)
- \( L \) = star’s luminosity, in \( \text{W} \)
- \( d \) = distance to star, in meters

This relationship is called the **inverse-square law**, because the apparent brightness of light that an observer can see or measure is inversely proportional to the square of the observer’s distance \((d) \) from the source. If you double your distance from a light source, its radiation is spread out over an area 4 times larger, so the apparent brightness you see is decreased by a factor of 4. Similarly, at triple the distance, the apparent brightness is \( 1/9 \) as great (see Figure 17-4).

We can apply the inverse-square law to the Sun, which is \( 1.50 \times 10^{11} \text{ m} \) from Earth. Its apparent brightness \((b_{\odot}) \) is

\[
b_{\odot} = \frac{3.86 \times 10^{26} \text{ W}}{4\pi(1.50 \times 10^{11} \text{ m})^2} = 1370 \text{ W/m}^2
\]
Calculating a Star’s Luminosity

The inverse-square law says that we can find a star’s luminosity if we know its distance and its apparent brightness. To do this, it is convenient to express this law in a somewhat different form. We first rearrange the above equation:

\[ L = 4\pi d^2 b \]

We then apply this equation to the Sun. That is, we write a similar equation relating the Sun’s luminosity \( L_\odot \), the distance from the Earth to the Sun \( d_\odot \), equal to 1 AU, and the Sun’s apparent brightness \( b_\odot \):

\[ L_\odot = 4\pi d_\odot^2 b_\odot \]
If we take the ratio of these two equations, the unpleasant factor of $4\pi$ drops out and we are left with the following:

**Determining a star’s luminosity from its apparent brightness**

$$\frac{L}{L_\odot} = \left(\frac{d}{d_\odot}\right)^2 \frac{b}{b_\odot}$$

$L/L_\odot$ = ratio of the star’s luminosity to the Sun’s luminosity  
$d/d_\odot$ = ratio of the star’s distance to the Earth-Sun distance  
$b/b_\odot$ = ratio of the star’s apparent brightness to the Sun’s apparent brightness

We need to know just two things to find a star’s luminosity: the distance to a star as compared to the Earth-Sun distance (the ratio $d/d_\odot$), and how that star’s apparent brightness compares to that of the Sun (the ratio $b/b_\odot$). Then we can use the above equation to find how luminous that star is compared to the Sun (the ratio $L/L_\odot$).

In other words, this equation gives us a general rule relating the luminosity, distance, and apparent brightness of a star:

**We can determine the luminosity of a star from its distance and apparent brightness. For a given distance, the brighter the star, the more luminous that star must be. For a given apparent brightness, the more distant the star, the more luminous it must be to be seen at that distance.**

Box 17-2 shows how to use the above equation to determine the luminosity of the nearby star $\epsilon$ (epsilon) Eridani, the fifth brightest star in the constellation Eridanus (named for a river in Greek mythology). Parallax measurements indicate that $\epsilon$ Eridani is 3.23 pc away, and photometry shows that the star appears only $6.73 \times 10^{-13}$ as bright as the Sun; using the above equation, we find that $\epsilon$ Eridani has only 0.30 times the luminosity of the Sun.

The Stellar Population

Calculations of this kind show that stars come in a wide variety of different luminosities, with values that range from about $10^6 \ L_\odot$ (a million times the Sun’s luminosity) to only about $10^{-4} \ L_\odot$ (a mere ten-thousandth of the Sun’s light output). The most luminous star emits roughly $10^{10}$ times more energy each second than the least luminous! (To put this number in perspective, about $10^{10}$ human beings have lived on the Earth since our species first evolved.)

As stars go, our Sun is neither extremely luminous nor extremely dim; it is a rather ordinary, garden-variety star. It is somewhat more luminous than most stars, however. Of more than 30 stars within 4 pc of the Sun (see Appendix 4), only three ($\alpha$ Centauri, Sirius, and Procyon) have a greater luminosity than the Sun.

To better characterize a typical population of stars, astronomers count the stars out to a certain distance from the Sun and plot the number of stars that have different luminosities. The resulting graph is called the **luminosity function**. Figure 17-5 shows the luminosity function for stars in our part of the Milky Way Galaxy. The curve declines very steeply for the most luminous stars toward the left side of the graph, indicating that they are quite rare. For example, this graph shows that stars like the Sun are about 10,000 times more common than stars like Spica (which has a luminosity of 2100 $L_\odot$).

The exact shape of the curve in Figure 17-5 applies only to the vicinity of the Sun and similar regions in our Milky Way Galaxy. Other locations have somewhat different luminosity functions. In stellar populations in general, however, low-luminosity stars are much more common than high-luminosity ones.

17-3 **Astronomers often use the magnitude scale to denote brightness**

Because astronomy is among the most ancient of sciences, some of the tools used by modern astronomers are actually many centuries old. One such tool is the **magnitude scale**, which astronomers frequently use to denote the brightness of stars. This scale was introduced in the second century B.C. by the Greek astronomer Hipparchus, who called the brightest stars first-magnitude stars. Stars about half as bright as first-magnitude stars were called second-magnitude stars, and so forth, down to sixth-magnitude stars, the dimmest ones he could see. After telescopes came into use, astronomers extended Hipparchus’s magnitude scale to include even dimmer stars.

**Apparent Magnitudes**

The magnitudes in Hipparchus’s scale are properly called **apparent magnitudes**, because they describe how bright an object appears to an Earth-based observer. Apparent magnitude is directly related to apparent brightness.
CAUTION! The magnitude scale can be confusing because it works “backward.” Keep in mind that the greater the apparent magnitude, the dimmer the star. A star of apparent magnitude +3 (a third-magnitude star) is dimmer than a star of apparent magnitude +2 (a second-magnitude star).

In the nineteenth century, astronomers developed better techniques for measuring the light energy arriving from a star. These measurements showed that a first-magnitude star is about 100 times brighter than a sixth-magnitude star. In other words, it would take 100 stars of magnitude +6 to provide as much light energy as we receive from a single star of magnitude +1. To make computations easier, the magnitude scale was redefined so that a magnitude difference of 5 corresponds exactly to a factor of 100 in brightness. A magnitude difference of 1 corresponds to a factor of 2.512 in brightness, because

\[2.512 \times 2.512 \times 2.512 \times 2.512 \times 2.512 = (2.512)^5 = 100\]

Thus, it takes 2.512 third-magnitude stars to provide as much light as we receive from a single second-magnitude star.

Figure 17-6 illustrates the modern apparent magnitude scale. The dimmest stars visible through a pair of binoculars have an apparent magnitude of +10, and the dimmest stars that can be photographed in a one-hour exposure with the Keck telescopes (see Section 6-2) or the Hubble Space Telescope have apparent magnitude +30. Modern astronomers also use negative numbers to extend Hipparchus’s scale to include very bright objects. For example, Sirius, the brightest star in the sky, has an apparent magnitude of −1.43. The Sun, the brightest object in the sky, has an apparent magnitude of −26.7.

**Absolute Magnitudes**

Apparent magnitude is a measure of a star’s apparent brightness as seen from Earth. A related quantity that measures a star’s true energy output—that is, its luminosity—is called absolute magnitude. This is the apparent magnitude a star would have if it were located exactly 10 parsecs from Earth.

**ANALOGY** If you wanted to compare the light output of two different lightbulbs, you would naturally place them side by side so that both bulbs were the same distance from you. In the absolute magnitude scale, we imagine doing the same thing with stars to compare their luminosities.

If the Sun were moved to a distance of 10 parsecs from the Earth, it would have an apparent magnitude of +4.8. The absolute magnitude of the Sun is thus +4.8. The absolute magnitudes

![Figure 17-6](https://example.com/figure17-6.png)

(a) Some apparent magnitudes

- Sun (−26.7)
- Full moon (−12.6)
- Venus (at brightest) (−4.4)
- Sirius (brightest star) (−1.4)
- Naked eye limit (+6.0)
- Binocular limit (+10.0)
- Pluto (+15.1)
- Large telescope (visual limit) (+21.0)
- Hubble Space Telescope and large Earth-based telescopes (photographic limit) (+30.0)

(b) Apparent magnitudes of stars in the Pleiades

![The Apparent Magnitude Scale](https://example.com/magnitude.png)

Astronomers denote the apparent brightness of objects in the sky by their apparent magnitudes. The greater the apparent magnitude, the dimmer the object. This photograph of the Pleiades cluster, located about 120 pc away in the constellation Taurus, shows the apparent magnitudes of some of its stars. Most are too faint to be seen by the naked eye.

(David Malin/Anglo-Australian Observatory)
Apparent Magnitude and Absolute Magnitude

Astronomers commonly express a star’s apparent brightness in terms of apparent magnitude (denoted by a lowercase \( m \)), and the star’s luminosity in terms of absolute magnitude (denoted by a capital \( M \)). While we do not use these quantities extensively in this book, it is useful to know a few simple relationships involving them.

Consider two stars, labeled 1 and 2, with apparent magnitudes \( m_1 \) and \( m_2 \) and brightnesses \( b_1 \) and \( b_2 \), respectively. The ratio of their apparent brightnesses \( \frac{b_1}{b_2} \) corresponds to a difference in their apparent magnitudes \( m_2 - m_1 \). As we learned in Section 17-3, each step in magnitude corresponds to a factor of 2.512 in brightness; we receive 2.512 times more energy per square meter per second from a third-magnitude star than from a fourth-magnitude star. This idea was used to construct the following table:

<table>
<thead>
<tr>
<th>Apparent magnitude difference ( (m_2 = m_1) )</th>
<th>Ratio of apparent brightness ( \frac{b_1}{b_2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.512</td>
</tr>
<tr>
<td>2</td>
<td>((2.512)^2 = 6.31)</td>
</tr>
<tr>
<td>3</td>
<td>((2.512)^3 = 15.85)</td>
</tr>
<tr>
<td>4</td>
<td>((2.512)^4 = 39.82)</td>
</tr>
<tr>
<td>5</td>
<td>((2.512)^5 = 100)</td>
</tr>
<tr>
<td>10</td>
<td>((2.512)^{10} = 10^{4})</td>
</tr>
<tr>
<td>15</td>
<td>((2.512)^{15} = 10^{6})</td>
</tr>
<tr>
<td>20</td>
<td>((2.512)^{20} = 10^{8})</td>
</tr>
</tbody>
</table>

A simple equation relates the difference between two stars’ apparent magnitudes to the ratio of their brightnesses:

**Magnitude difference related to brightness ratio**

\[
m_2 - m_1 = 2.5 \log \left( \frac{b_1}{b_2} \right)
\]

\( m_1, m_2 \) = apparent magnitudes of stars 1 and 2

\( b_1, b_2 \) = apparent brightnesses of stars 1 and 2

In this equation, \( \log \left( \frac{b_1}{b_2} \right) \) is the logarithm of the brightness ratio. The logarithm of 1000 = \( 10^3 \) is 3, the logarithm of 10 = \( 10^1 \) is 1, and the logarithm of 1 = \( 10^0 \) is 0.

**EXAMPLE:** At their most brilliant, Venus has a magnitude of about \(-4\) and Mercury has a magnitude of about \(-2\). How many times brighter are these than the dimmest stars visible to the naked eye, with a magnitude of \(+6\)?
The inverse-square law relating a star’s apparent brightness and luminosity can be rewritten in terms of the star’s apparent magnitude \(m\), absolute magnitude \(M\), and distance from the Earth \(d\). This can be expressed as an equation:

**Relation between a star’s apparent magnitude and absolute magnitude**

\[
m - M = 5 \log d - 5
\]

\(m\) = star’s apparent magnitude \\
\(M\) = star’s absolute magnitude \\
\(d\) = distance from the Earth to the star in parsecs

In this expression \(m - M\) is called the **distance modulus**, and \(\log d\) means the logarithm of the distance \(d\) in parsecs. For convenience, the following table gives the values of the distance \(d\) corresponding to different values of \(m - M\).

<table>
<thead>
<tr>
<th>Distance modulus (m - M)</th>
<th>Distance (d) (pc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>1.6</td>
</tr>
<tr>
<td>-3</td>
<td>2.5</td>
</tr>
<tr>
<td>-2</td>
<td>4.0</td>
</tr>
<tr>
<td>-1</td>
<td>6.3</td>
</tr>
<tr>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>16</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
</tr>
<tr>
<td>4</td>
<td>63</td>
</tr>
<tr>
<td>5</td>
<td>100</td>
</tr>
<tr>
<td>10</td>
<td>(10^3)</td>
</tr>
<tr>
<td>15</td>
<td>(10^4)</td>
</tr>
<tr>
<td>20</td>
<td>(10^5)</td>
</tr>
</tbody>
</table>

This table shows that if a star is less than 10 pc away, its distance modulus \(m - M\) is negative. That is, its apparent magnitude \(m\) is less than its absolute magnitude \(M\). If the star is more than 10 pc away, \(m - M\) is positive and \(m\) is greater than \(M\). As an example, the star \(\epsilon\) (epsilon) Indi, which is in the direction of the southern constellation Indus, has apparent magnitude \(m = +4.7\). It is 3.6 pc away, which is less than 10 pc, so its apparent magnitude is less than its absolute magnitude.

**EXAMPLE:** Find the absolute magnitude of \(\epsilon\) Indi.

**Situation:** We are given the distance to \(\epsilon\) Indi \((d = 3.6\) pc\) and its apparent magnitude \((m = +4.7)\). Our goal is to find the star’s absolute magnitude \(M\).

**Tools:** We use the formula \(m - M = 5 \log d - 5\) to solve for \(M\).

**Answer:** Since \(d = 3.6\) pc, we use a calculator to find \(\log d = \log 3.6 = 0.56\). Therefore the star’s distance modulus is \(m - M = 5(0.56) - 5 = -2.2\), and the star’s absolute magnitude is \(M = m - (-2.2) = +4.7 + 2.2 = +6.9\).

**Review:** As a check on our calculations, note that this star’s distance modulus \(m - M = -2.2\) is less than zero, as it should be for a star less than 10 pc away. Note that our Sun has absolute magnitude \(+4.8\); \(\epsilon\) Indi has a greater absolute magnitude, so it is less luminous than the Sun.

**EXAMPLE:** Suppose you were viewing the Sun from a planet orbiting another star 100 pc away. Could you see it without using a telescope?

**Situation:** We learned in the preceding examples that the Sun has absolute magnitude \(M = +4.8\) and that the dimmest stars visible to the naked eye have apparent magnitude \(m = +6\). Our goal is to determine whether the Sun would be visible to the naked eye at a distance of 100 pc.

**Tools:** We use the relationship \(m - M = 5 \log d - 5\) to find the Sun’s apparent magnitude at \(d = 100\) pc. If this is greater than \(+6\), the Sun would not be visible at that distance. (Remember that the greater the apparent magnitude, the dimmer the star.)

**Answer:** From the table, at \(d = 100\) pc the distance modulus is \(m - M = 5\). So, as seen from this distant planet, the Sun’s apparent magnitude would be \(m = M + 5 = +4.8 + 5 = +9.8\). This is greater than the naked-eye limit \(m = +6\), so the Sun could not be seen.

**Review:** The Sun is by far the brightest object in the Earth’s sky. But our result tells us that to an inhabitant of a planetary system 100 pc away—a rather small distance in a galaxy that is thousands of parsecs across—our own Sun would be just another insignificant star, visible only through a telescope.

The magnitude system is also used by astronomers to express the colors of stars as seen through different filters, as we describe in Section 17-4. For example, rather than quantifying a star’s color by the color ratio \(b_v/b_B\) (a star’s apparent brightness as seen through a V filter divided by the brightness through a B filter), astronomers commonly use the **color index** \(B-V\), which is the difference in the star’s apparent magnitude as measured with these two filters. We will not use this system in this book, however (but see Advanced Questions 51 and 52).
of the stars range from approximately $+15$ for the least luminous to $-10$ for the most luminous. (Note: Like apparent magnitudes, absolute magnitudes work “backward”: The greater the absolute magnitude, the less luminous the star.) The Sun’s absolute magnitude is about in the middle of this range.

We saw in Section 17-2 that we can calculate the luminosity of a star if we know its distance and apparent brightness. There is a mathematical relationship between absolute magnitude and luminosity, which astronomers use to convert one to the other as they see fit. It is also possible to rewrite the inverse-square law, which we introduced in Section 17-2, as a mathematical relationship that allows you to calculate a star’s absolute magnitude (a measure of its luminosity) from its distance and apparent magnitude (a measure of its apparent brightness). Box 17-3 describes these relationships and how to use them.

Because the “backward” magnitude scales can be confusing, we will use them only occasionally in this book. We will usually speak of a star’s luminosity rather than its absolute magnitude and will describe a star’s appearance in terms of apparent brightness rather than apparent magnitude. But if you go on to study more about astronomy, you will undoubtedly make frequent use of apparent magnitude and absolute magnitude.

**17-4 A star’s color depends on its surface temperature**

The image that opens this chapter shows that stars come in different colors. You can see these colors even with the naked eye. For example, you can easily see the red color of Betelgeuse, the star in the “armpit” of the constellation Orion, and the blue tint of Bellatrix at Orion’s other “shoulder” (see Figure 2-2). Colors are most evident for the brightest stars, because your color vision does not work well at low light levels.

**CAUTION!** It’s true that the light from a star will appear red-shifted if the star is moving away from you and blueshifted if it’s moving toward you. But for even the fastest stars, these color shifts are so tiny that it takes sensitive instruments to measure them. The red color of Betelgeuse and the blue color of Bellatrix are not due to their motions; they are the actual colors of the stars.

**Color and Temperature**

We saw in Section 5-3 that a star’s color is directly related to its surface temperature. The intensity of light from a relatively cool star peaks at long wavelengths, making the star look red (Figure 17-7a). A hot star’s intensity curve peaks at shorter wavelengths, so the star looks blue (Figure 17-7c). For a star with an intermediate temperature, such as the Sun, the intensity peak is near the middle of the visible spectrum. This gives the star a yellowish color (Figure 17-7b). This leads to an important general rule about star colors and surface temperatures:

*Red stars are relatively cold, with low surface temperatures; blue stars are relatively hot, with high surface temperatures.*

Figure 17-7 shows that astronomers can accurately determine the surface temperature of a star by carefully measuring its color.
Temperature.

Comparing the results, an astronomer can determine the star's surface temperature. To do this, the star’s light is collected by a telescope and passed through one of a set of differently colored filters. The filtered light is then collected by a light-sensitive device such as a CCD (see Section 6-4). The process is then repeated with each of the filters in the set. The star’s image will have a different brightness through each colored filter, and by comparing these brightnesses astronomers can find the wavelength at which the star’s intensity curve has its peak—and hence the star’s temperature.

**UBV Photometry**

Let’s look at this procedure in more detail. The most commonly used filters are called U, B, and V, and the technique that uses them is called UBV photometry. Each filter is transparent in a different band of wavelengths: the ultraviolet (U), the blue (B), and the yellow-green (V, for visual) region of the visible spectrum (Figure 17-8). The transparency of the V filter mimics the sensitivity of the human eye.

To determine a star’s temperature using UBV photometry, the astronomer first measures the star’s brightness through each of the filters individually. This gives three apparent brightnesses for the star, designated \( b_U \), \( b_B \), and \( b_V \). The astronomer then compares the intensity of starlight in neighboring wavelength bands by taking the ratios of these brightnesses: \( b_v/b_b \) and \( b_b/b_U \). Table 17-1 gives values for these color ratios for several stars with different surface temperatures.

If a star is very hot, its radiation is skewed toward short, ultraviolet wavelengths as in Figure 17-7c. This makes the star dim through the V filter, brighter through the B filter, and brightest through the U filter. Hence, for a hot star \( b_V \) is less than \( b_B \), which in turn is less than \( b_U \), and the ratios \( b_V/b_B \) and \( b_B/b_U \) are both less than 1. One such star is Bellatrix (see Table 17-1), which has a surface temperature of 21,500 K.

In contrast, if a star is cool, its radiation peaks at long wavelengths as in Figure 17-7a. Such a star appears brightest through the V filter, dimmer through the B filter, and dimmest through the U filter (see Figure 17-8). In other words, for a cool star \( b_V \) is greater than \( b_B \), which in turn is greater than \( b_U \). Hence, the ratios \( b_V/b_B \) and \( b_B/b_U \) will both be greater than 1. The star Betelgeuse (surface temperature 3500 K) is an example.

You can see these differences between hot and cool stars in parts a and c of Figure 6-30, which show the constellation Orion at ultraviolet wavelengths (a bit shorter than those transmitted by the U filter) and at visible wavelengths that approximate the transmission of a V filter. The hot star Bellatrix is brighter in the ultraviolet image (Figure 6-30a) than at visible wavelengths (Figure 6-30c). (Figure 6-30d shows the names of the stars.) The situation is reversed for the cool star Betelgeuse: It is bright at visible wavelengths, but at ultraviolet wavelengths it is too dim to show up in the image.

Figure 17-9 graphs the relationship between a star’s \( b_V/b_B \) color ratio and its temperature. If you know the value of the \( b_V/b_B \) color ratio for a given star, you can use this graph to find the star’s surface temperature. As an example, for the Sun \( b_V/b_B \)

<table>
<thead>
<tr>
<th>Star</th>
<th>Surface temperature (K)</th>
<th>( b_V/b_B )</th>
<th>( b_B/b_U )</th>
<th>Apparent color</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bellatrix (γ Orionis)</td>
<td>21,500</td>
<td>0.81</td>
<td>0.45</td>
<td>Blue</td>
</tr>
<tr>
<td>Regulus (α Leonis)</td>
<td>12,000</td>
<td>0.90</td>
<td>0.72</td>
<td>Blue-white</td>
</tr>
<tr>
<td>Sirius (α Canis Majoris)</td>
<td>9400</td>
<td>1.00</td>
<td>0.96</td>
<td>Blue-white</td>
</tr>
<tr>
<td>Megrez (δ Ursae Majoris)</td>
<td>8630</td>
<td>1.07</td>
<td>1.07</td>
<td>White</td>
</tr>
<tr>
<td>Altair (α Aquilae)</td>
<td>7800</td>
<td>1.23</td>
<td>1.08</td>
<td>Yellow-white</td>
</tr>
<tr>
<td>Sun</td>
<td>5800</td>
<td>1.87</td>
<td>1.17</td>
<td>Yellow-white</td>
</tr>
<tr>
<td>Aldebaran (α Tauri)</td>
<td>4000</td>
<td>4.12</td>
<td>5.76</td>
<td>Orange</td>
</tr>
<tr>
<td>Betelgeuse (α Orionis)</td>
<td>3500</td>
<td>5.55</td>
<td>6.66</td>
<td>Red</td>
</tr>
</tbody>
</table>

Source: J.-C. Mermilliod, B. Hauck, and M. Mermilliod, University of Lausanne
We have seen how the color of a star’s light helps astronomers determine its surface temperature. To determine the other properties of a star, astronomers must analyze the spectrum of its light. This technique of stellar spectroscopy began in 1817 when Joseph Fraunhofer, a German instrument maker, attached a spectroscope to a telescope and pointed it toward the stars. Fraunhofer had earlier observed that the Sun has an absorption line spectrum—that is, a continuous spectrum with dark absorption lines (see Section 5-6). He found that stars have the same kind of spectra, which reinforces the idea that our Sun is a rather typical star. But Fraunhofer also found that the pattern of absorption lines is different for different stars.

Some stars have spectra in which the Balmer absorption lines of hydrogen are prominent. But in the spectra of other stars, including the Sun, the Balmer lines are nearly absent and the dominant absorption lines are those of heavier elements such as calcium, iron, and sodium. Still other stellar spectra are dominated by broad absorption lines caused by molecules, such as titanium oxide, rather than single atoms. To cope with this diversity, astronomers group similar-appearing stellar spectra into spectral classes. In a popular classification scheme that emerged in the late 1890s, a star was assigned a letter from A through O according to the strength or weakness of the hydrogen Balmer lines in the star’s spectrum.

Nineteenth-century science could not explain why or how the spectral lines of a particular chemical are affected by the temperature and density of the gas. Nevertheless, a team of astronomers at the Harvard College Observatory forged ahead with a monumental project of examining the spectra of hundreds of thousands of stars. Their goal was to develop a system of spectral classification in which all spectral features, not just Balmer lines, change smoothly from one spectral class to the next.

The Harvard project was financed by the estate of Henry Draper, a wealthy New York physician and amateur astronomer who in 1872 became the first person to photograph stellar absorption lines. Researchers on the project included Edward C. Pickering, Willaimina Fleming, Antonia Maury, and Annie Jump Cannon (Figure 17-10). As a result of their efforts, many of the original A-through-O classes were dropped and others were consolidated. The remaining spectral classes were reordered in the sequence OBAGFKM. You can remember this sequence with the mnemonic: “Oh, Be A Fine Girl (or Guy), Kiss Me!”

Classifying Stars: Absorption Line Spectra and Spectral Classes

We see an absorption line spectrum when a cool gas lies between us and a hot, glowing object (recall Figure 5-16). The light from the hot, glowing object itself has a continuous spectrum. In the case of a star, light with a continuous spectrum is produced at low-lying levels of the star’s atmosphere where the gases are hot and dense. The absorption lines are created when this light flows outward through the upper layers of the star’s atmosphere. Atoms in these cooler, less dense layers absorb radiation at specific wavelengths, which depend on the specific kinds of atoms present—hydrogen, helium, or other elements—and on whether or not the atoms are ionized. Absorption lines in the Sun’s spectrum are produced in this same way (see Section 16-5).

Deciphering the information in starlight took the painstaking work of generations of astronomers
Why Surface Temperature Affects Stellar Spectra

To see why the appearance of a star’s spectrum is profoundly affected by the star’s surface temperature, consider the Balmer lines of hydrogen. Hydrogen is by far the most abundant element in the universe, accounting for about three-quarters of the mass of a typical star. Yet the Balmer lines do not necessarily show up in a star’s spectrum. As we saw in Section 5-8, Balmer absorption lines are produced when an electron in the \( n = 2 \) orbit of hydrogen is lifted into a higher orbit by absorbing a photon with the right amount of energy (see Figure 5-22). If the star is much hotter than 10,000 K, the photons pouring out of the star’s interior have such high energy that they easily knock electrons out of hydrogen atoms in the star’s atmosphere. This process ionizes the gas. With its only electron torn away, a hydrogen atom cannot produce absorption lines. Hence, the Balmer lines will be relatively weak in the spectra of such hot stars, such as the hot O and B2 stars in Figure 17-11.

Conversely, if the star’s atmosphere is much cooler than 10,000 K, almost all the hydrogen atoms are in the lowest \( (n = 1) \) energy state. Most of the photons passing through the star’s atmosphere possess too little energy to boost electrons up from the \( n = 1 \) to the \( n = 2 \) orbit of the hydrogen atoms. Hence, very few of these atoms will have electrons in the \( n = 2 \) orbit, and only these few can absorb the photons characteristic of the Balmer lines. As a result, these lines are nearly absent from the spectrum of a cool star. (You can see this in the spectra of the cool M0 and M2 stars in Figure 17-11.)

For the Balmer lines to be prominent in a star’s spectrum, the star must be hot enough to excite the electrons out of the ground state but not so hot that all the hydrogen atoms become ionized. A stellar surface temperature of about 9000 K produces the strongest hydrogen lines; this is the case for the stars of spectral types A0 and A5 in Figure 17-11.

Every other type of atom or molecule also has a characteristic temperature range in which it produces prominent absorption lines in the observable part of the spectrum. Figure 17-12 shows the relative strengths of absorption lines produced by different chemicals. By measuring the details of these lines in a given star’s spectrum, astronomers can accurately determine that star’s surface temperature.

For example, the spectral lines of neutral (that is, un-ionized) helium are strong around 25,000 K. At this temperature, photons have enough energy to excite helium atoms without tearing away the electrons altogether. In stars hotter than about 30,000 K, helium atoms become singly ionized, that is, they lose one of their two electrons. The remaining electron produces a set of spectral lines that is recognizably different from those of neutral helium. Hence, when the spectral lines of singly ionized helium appear in a star’s spectrum, we know that the star’s surface temperature is greater than 30,000 K.

Astronomers use the term metals to refer to all elements other than hydrogen and helium. (This idiosyncratic use of the term “metal” is quite different from the definition used by chemists and other scientists. To a chemist, sodium and iron are metals but carbon and oxygen are not; to an astronomer, all of these substances are metals.) In this terminology, metals dominate the
Chapter 17

Figure 17-11  Principal Types of Stellar Spectra
Stars of different spectral classes and different surface temperatures have spectra dominated by different absorption lines. Notice how the Balmer lines of hydrogen (H\textsubscript{α}, H\textsubscript{β}, H\textsubscript{γ}, and H\textsubscript{δ}) are strongest for hot stars of spectral class A, while absorption lines due to calcium (Ca) are strongest in medium-temperature G and K stars. The spectra of M stars have broad, dark bands caused by molecules of titanium oxide (TiO), which can only exist at relatively low temperatures. A roman numeral after a chemical symbol shows whether the absorption line is caused by un-ionized atoms (roman numeral I) or by atoms that have lost one electron (roman numeral II). (R. Bell, University of Maryland, and M. Briley, University of Wisconsin at Oshkosh)

Figure 17-12  The Strengths of Absorption Lines
Each curve in this graph peaks at the stellar surface temperature for which that chemical's absorption line is strongest. For example, hydrogen (H) absorption lines are strongest in A stars with surface temperatures near 10,000 K. Roman numeral I denotes neutral, un-ionized atoms; II, III, and IV denote atoms that are singly, doubly, or triply ionized (that is, have lost one, two, or three electrons).
spectra of stars cooler than 10,000 K. Ionized metals are prominent for surface temperatures between 6000 and 8000 K, while neutral metals are strongest between approximately 5500 and 4000 K.

Below 4000 K, certain atoms in a star’s atmosphere combine to form molecules. (At higher temperatures atoms move so fast that when they collide, they bounce off each other rather than “sticking together” to form molecules.) As these molecules vibrate and rotate, they produce bands of spectral lines that dominate the star’s spectrum. Most noticeable are the lines of titanium oxide (TiO), which are strongest for surface temperatures of about 3000 K.

### Spectral Classes for Brown Dwarfs

Since 1995 astronomers have found a number of stars with surface temperatures even lower than those of spectral class M. Strictly speaking, these are not stars but brown dwarfs, which we introduced in Section 8-6. Brown dwarfs are too small to sustain thermonuclear fusion in their cores. Instead, these “substars” glow primarily from the heat released by Kelvin-Helmholtz contraction, which we described in Section 16-1. (They do undergo fusion reactions for a brief period during their evolution.) Brown dwarfs are so cold that they are best observed with infrared telescopes (see Figure 17-13). Such observations reveal that brown dwarf spectra have a rich variety of absorption lines due to molecules. Some of these molecules actually form into solid grains in a brown dwarf’s atmosphere.

To describe brown dwarf spectra, astronomers have defined two new spectral classes, L and T. Thus, the modern spectral sequence of stars and brown dwarfs from hottest to coldest surface temperature is OBAFGKMLT. (Can you think of a new mnemonic that includes L and T?) For example, Figure 17-13 shows a star of spectral class K and a brown dwarf of spectral class T. Table 17-2 summarizes the relationship between the temperature and spectra of stars and brown dwarfs.

### Table 17-2 The Spectral Sequence

<table>
<thead>
<tr>
<th>Spectral class</th>
<th>Color</th>
<th>Temperature (K)</th>
<th>Spectral lines</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>Blue-violet</td>
<td>30,000–50,000</td>
<td>Ionized atoms, especially helium</td>
<td>Naos (ξ Puppis), Mintaka (δ Orionis)</td>
</tr>
<tr>
<td>B</td>
<td>Blue-white</td>
<td>11,000–30,000</td>
<td>Neutral helium, some hydrogen</td>
<td>Spica (α Virginis), Rigel (β Orionis)</td>
</tr>
<tr>
<td>A</td>
<td>White</td>
<td>7500–11,000</td>
<td>Strong hydrogen, some ionized metals</td>
<td>Sirius (α Canis Majoris), Vega (α Lyrae)</td>
</tr>
<tr>
<td>F</td>
<td>Yellow-white</td>
<td>5900–7500</td>
<td>Hydrogen and ionized metals such as calcium and iron</td>
<td>Canopus (α Carinae), Procyon (α Canis Minoris)</td>
</tr>
<tr>
<td>G</td>
<td>Yellow</td>
<td>5200–5900</td>
<td>Both neutral and ionized metals, especially ionized calcium</td>
<td>Sun, Capella (α Aurigae)</td>
</tr>
<tr>
<td>K</td>
<td>Orange</td>
<td>3900–5200</td>
<td>Neutral metals</td>
<td>Arcturus (α Boötis), Aldebaran (α Tauri)</td>
</tr>
<tr>
<td>M</td>
<td>Red-orange</td>
<td>2500–3900</td>
<td>Strong titanium oxide and some neutral calcium</td>
<td>Antares (α Scorpii), Betelgeuse (α Orionis)</td>
</tr>
<tr>
<td>L</td>
<td>Red</td>
<td>1300–2500</td>
<td>Neutral potassium, rubidium, and cesium, and metal hydrides</td>
<td>Brown dwarf Teide 1</td>
</tr>
<tr>
<td>T</td>
<td>Red</td>
<td>below 1300</td>
<td>Strong neutral potassium and some water (H₂O)</td>
<td>Brown dwarfs Gliese 229B, HD 3651B</td>
</tr>
</tbody>
</table>
Stellar Radii, Luminosities, and Surface Temperatures

Because stars emit light in almost exactly the same fashion as blackbodies, we can use the Stefan-Boltzmann law to relate a star’s luminosity ($L$), surface temperature ($T$), and radius ($R$). The relevant equation is

$$L = 4\pi R^2\sigma T^4$$

As written, this equation involves the Stefan-Boltzmann constant $\sigma$, which is equal to $5.67 \times 10^{-8}$ W m$^{-2}$ K$^{-4}$. In many calculations, it is more convenient to relate everything to the Sun, which is a typical star. Specifically, for the Sun we have $L_\odot = 4\pi R_\odot^2\sigma T_\odot^4$, where $L_\odot$ is the Sun’s luminosity, $R_\odot$ is the Sun’s radius, and $T_\odot$ is the Sun’s surface temperature (equal to 5800 K). Dividing the general equation for $L$ by this specific equation for the Sun, we obtain

$$\frac{L}{L_\odot} = \left(\frac{R}{R_\odot}\right)^2 \left(\frac{T}{T_\odot}\right)^4$$

This is an easier formula to use because the constant $\sigma$ has cancelled out. We can also rearrange terms to arrive at a useful equation for the radius ($R$) of a star:

$$\frac{R}{R_\odot} = \left(\frac{T_\odot}{T}\right)^{\frac{2}{4}} \left(\frac{L}{L_\odot}\right)^{1/4}$$

$$R/R_\odot = \text{ratio of the star’s radius to the Sun’s radius}$$

$$T_\odot/T = \text{ratio of the Sun’s surface temperature to the star’s surface temperature}$$

$$L/L_\odot = \text{ratio of the star’s luminosity to the Sun’s luminosity}$$

EXAMPLE: The bright reddish star Betelgeuse in the constellation Orion (see Figure 2-2 or Figure 6-30c) is 60,000 times more luminous than the Sun and has a surface temperature of 3500 K. What is its radius?

**Situation:** We are given the star’s luminosity $L = 60,000 L_\odot$ and its surface temperature $T = 3500$ K. Our goal is to find the star’s radius $R$.

When the effects of temperature are accounted for, astronomers find that all stars have essentially the same chemical composition. We can state the results as a general rule:

**By mass, almost all stars (including the Sun) and brown dwarfs are about three-quarters hydrogen, one-quarter helium, and 1% or less metals.**

Our Sun is about 1% metals by mass, as are most of the stars you can see with the naked eye. But some stars have an even lower percentage of metals. We will see in Chapter 19 that these seemingly minor differences tell an important tale about the life stories of stars.

Stars come in a wide variety of sizes

With even the best telescopes, stars appear as nothing more than bright points of light. On a photograph or CCD image, brighter stars appear larger than dim ones (see Figures 17-3, 17-6b, and 17-13), but these apparent sizes give no indication of the star’s actual size. To determine the size of a star, astronomers combine information about its luminosity (determined from its distance and apparent brightness) and its surface temperature (determined from its spectral type). In this way, they find that some stars are quite a bit smaller than the Sun, while others are a thousand times larger.

Calculating the Radii of Stars

The key to finding a star’s radius from its luminosity and surface temperature is the Stefan-Boltzmann law (see Section 5-4). This law says that the amount of energy radiated per second from a square meter of a blackbody’s surface—that is, the energy flux ($F$)—is proportional to the fourth power of the temperature of that surface ($T$), as given by the equation $F = \sigma T^4$. This equation applies very well to stars, whose spectra are quite similar to that of a perfect blackbody. (Absorption lines, while important for determining the star’s chemical composition and surface temperature, make only relatively small modifications to a star’s blackbody spectrum.)

A star’s luminosity is the amount of energy emitted per second from its entire surface. This equals the energy flux $F$ multiplied by the total number of square meters on the star’s surface (that is, the star’s surface area). We expect that most stars are nearly spherical, like the Sun, so we can use the formula for the surface area of a sphere. This is $4\pi R^2$, where $R$ is the star’s
radius (the distance from its center to its surface). Multiplying together the formulas for energy flux and surface area, we can write the star’s luminosity as follows:

**Relationship between a star’s luminosity, radius, and surface temperature**

\[ L = 4\pi R^2 \sigma T^4 \]

- \( L \) = star’s luminosity, in watts
- \( R \) = star’s radius, in meters
- \( \sigma \) = Stefan-Boltzmann constant = \( 5.67 \times 10^{-8} \) W m\(^{-2}\) K\(^{-4}\)
- \( T \) = star’s surface temperature, in kelvins

This equation says that a relatively cool star (low surface temperature \( T \)), for which the energy flux is quite low, can nonetheless be very luminous if it has a large enough radius \( R \). Alternatively, a relatively hot star (large \( T \)) can have a very low luminosity if the star has only a little surface area (small \( R \)).

**Box 17-4** describes how to use the above equation to calculate a star’s radius if its luminosity and surface temperature are known. We can express the idea behind these calculations in terms of the following general rule:

**We can determine the radius of a star from its luminosity and surface temperature. For a given luminosity, the greater the surface temperature, the smaller the radius must be. For a given surface temperature, the greater the luminosity, the larger the radius must be.**

**ANALOGY.** In a similar way, a roaring campfire can emit more light than a welder’s torch. The campfire is at a lower temperature than the torch, but has a much larger surface area from which it emits light.

**The Range of Stellar Radii**

Using this general rule as shown in Box 17-4, astronomers find that stars come in a wide range of sizes. The smallest stars visible through ordinary telescopes, called white dwarfs, are about the same size as the Earth. Although their surface temperatures can be very high (25,000 K or more), white dwarfs have so little surface area that their luminosities are very low (less than 0.01 \( L_\odot \)). The largest stars, called supergiants, are a thousand times larger in radius than the Sun and \( 10^5 \) times larger than white dwarfs. If our own Sun were replaced by one of these supergiants, the Earth’s orbit would lie completely inside the star!

**Figure 17-14** summarizes how astronomers determine the distance from Earth, luminosity, surface temperature, chemical
composition, and radius of a star close enough to us so that its parallax can be measured. Remarkably, all of these properties can be deduced from just a few measured quantities: the star’s parallax angle, apparent brightness, and spectrum.

17-7 Hertzsprung-Russell (H-R) diagrams reveal the different kinds of stars

Astronomers have collected a wealth of data about the stars, but merely having tables of numerical data is not enough. Like all scientists, astronomers want to analyze their data to look for trends and underlying principles. One of the best ways to look for trends in any set of data, whether it comes from astronomy, finance, medicine, or meteorology, is to create a graph showing how one quantity depends on another. For example, investors consult graphs of stock market values versus dates, and weather forecasters make graphs of temperature versus altitude to determine whether thunderstorms will form. Astronomers have found that a particular graph of stellar properties shows that stars fall naturally into just a few categories. This graph, one of the most important in all astronomy, will in later chapters help us understand how stars form, evolve, and eventually die.

H-R Diagrams

Which properties of stars should we include in a graph? Most stars have about the same chemical composition, but two properties of stars—their luminosities and surface temperatures—differ substantially from one star to another. Stars also come in a wide range of radii, but a star’s radius is a secondary property that can be found from the luminosity and surface temperature (as we saw in Section 17-6 and Box 17-4). We also relegate the positions and space velocities of stars to secondary importance. (In a similar way, a physician is more interested in your weight and blood pressure than in where you live or how fast you drive.) We can then ask the following question: What do we learn when we graph the luminosities of stars versus their surface temperatures?

The first answer to this question was given in 1911 by the Danish astronomer Ejnar Hertzsprung. He pointed out that a regular pattern appears when the absolute magnitudes of stars (which measure their luminosities) are plotted against their colors (which measure their surface temperatures). Two years later, the American astronomer Henry Norris Russell independently discovered a similar regularity in a graph using spectral types (another measure of surface temperature) instead of colors. In recognition of their originators, graphs of this kind are today known as **Hertzsprung-Russell diagrams**, or **H-R diagrams** (Figure 17-15).

Figure 17-15a is a typical Hertzsprung-Russell diagram. Each dot represents a star whose spectral type and luminosity have been determined. The most luminous stars are near the top of the diagram, the least luminous stars near the bottom. Hot stars of spectral classes O and B are toward the left side of the graph and cool stars of spectral class M are toward the right.

**CAUTION!** You are probably accustomed to graphs in which the numbers on the horizontal axis increase as you move to the right. (For example, the business section of a newspaper includes a graph of stock market values versus dates, with later
But on an H-R diagram the temperature scale on the horizontal axis increases toward the left. This practice stems from the original diagrams of Hertzsprung and Russell, who placed hot O stars on the left and cool M stars on the right. This arrangement is a tradition that no one has seriously tried to change.

Star Varieties: Main-Sequence Stars, Giants, Supergiants, White Dwarfs, and Brown Dwarfs

The most striking feature of the H-R diagram is that the data points are not scattered randomly over the graph but are grouped in a few distinct regions. The luminosities and surface temperatures of stars do not have random values; instead, these two quantities are related!

The band stretching diagonally across the H-R diagram includes about 90% of the stars in the night sky. This band, called the main sequence, extends from the hot, luminous, blue stars in the upper left corner of the diagram to the cool, dim, red stars in the lower right corner. A star whose properties place it in this region of an H-R diagram is called a main-sequence star. The Sun (spectral type G2, luminosity $1 \ L_\odot$, absolute magnitude $-4.8$) is such a star. We will find that all main-sequence stars are like the Sun in that hydrogen fusion—thermonuclear reactions that convert hydrogen into helium (see Section 16-1)—is taking place in their cores.

The upper right side of the H-R diagram shows a second major grouping of data points. Stars represented by these points are both luminous and cool. From the Stefan-Boltzmann law, we
You can think of white dwarfs as “has-been” stars whose days of glory have passed. In this analogy, a brown dwarf is a “never-will-be.”

The existence of fundamentally different types of stars is the first important lesson to come from the H-R diagram. In later chapters we will find that these different types represent various stages in the lives of stars. We will use the H-R diagram as an essential tool for understanding how stars evolve.
Fundamentally, these differences between stars of different luminosity are due to differences between the stars’ atmospheres, where absorption lines are produced. Hydrogen lines in particular are affected by the density and pressure of the gas in a star’s atmosphere. The higher the density and pressure, the more frequently hydrogen atoms collide and interact with other atoms and ions in the atmosphere. These collisions shift the energy levels in the hydrogen atoms and thus broaden the hydrogen spectral lines.

In the atmosphere of a luminous giant star, the density and pressure are quite low because the star’s mass is spread over a huge volume. Atoms and ions in the atmosphere are relatively far apart; hence, collisions between them are inefficiently frequent so that hydrogen atoms can produce narrow Balmer lines. A main-sequence star, however, is much more compact than a giant or supergiant. In the denser atmosphere of a main-sequence star, frequent interatomic collisions perturb the energy levels in the hydrogen atoms, thereby producing broader Balmer lines.

**Luminosity Classes**

In the 1930s, W. W. Morgan and P. C. Keenan of the Yerkes Observatory of the University of Chicago developed a system of luminosity classes based upon the subtle differences in spectral lines. When these luminosity classes are plotted on an H-R diagram (Figure 17-17), they provide a useful subdivision of the star types in the upper right of the diagram. Luminosity classes Ia and Ib are composed of supergiants; luminosity class V includes all the main-sequence stars. The intermediate classes distinguish giant stars of various luminosities. Note that for stars of a given surface temperature (that is, a given spectral type), the higher the number of the luminosity class, the lower the star’s luminosity.

As we will see in Chapters 19 and 20, different luminosity classes represent different stages in the evolution of a star. White dwarfs are not given a luminosity class of their own; as we mentioned in Section 17-7, they represent a final stage in stellar evolution in which no thermonuclear reactions take place.

Astronomers commonly use a shorthand description that combines a star’s spectral type and its luminosity class. For example, the Sun is said to be a G2 V star. The spectral type indicates the star’s surface temperature, and the luminosity class indicates its luminosity. Thus, an astronomer knows immediately that any G2 V star is a main-sequence star with a luminosity of about 1 L\(_\odot\) and a surface temperature of about 5800 K. Similarly, a description of Aldebaran as a K5 III star tells an astronomer that it is a red giant with a luminosity of around 370 L\(_\odot\) and a surface temperature of about 4000 K.

**Spectroscopic Parallax**

A star’s spectral type and luminosity class, combined with the information on the H-R diagram, enable astronomers to estimate the star’s distance from the Earth. As an example, consider the star Pleione in the constellation Taurus. Its spectrum reveals Pleione to be a B8 V star (a hot, blue, main-sequence star, like the one in Figure 17-16b). Using Figure 17-17, we can read off that such a star’s luminosity is 190 L\(_\odot\). Given the star’s luminosity and its apparent brightness—in the case of Pleione, \(3.9 \times 10^{-13}\) of the apparent brightness of the Sun—we can use the inverse-square law to determine its distance from the Earth. The mathematical details are worked out in Box 17-2.

This method for determining distance, in which the luminosity of a star is found using spectroscopy, is called spectroscopic parallax. Figure 17-18 summarizes the method of spectroscopic parallax.

**CAUTION!** The name “spectroscopic parallax” is a bit misleading, because no parallax angle is involved! The idea is that measuring the star’s spectrum takes the place of measuring its parallax as a way to find the star’s distance. A better name for this method, although not the one used by astronomers, would be “spectroscopic distance determination.”

Spectroscopic parallax is an incredibly powerful technique. No matter how remote a star is, this technique allows astronomers...
to determine its distance, provided only that its spectrum and apparent brightness can be measured. Box 17-2 gives an example of how spectroscopic parallax has been used to find the distance to stars in other galaxies tens of millions of parsecs away. By contrast, we saw in Section 17-1 that “real” stellar parallaxes can be measured only for stars within a few hundred parsecs.

Unfortunately, spectroscopic parallax has its limitations; distances to individual stars determined using this method are only accurate to at best 10%. The reason is that the luminosity classes shown in Figure 17-17 are not thin lines on the H-R diagram but are moderately broad bands. Hence, even if a star’s spectral type and luminosity class are known, there is still some uncertainty in the luminosity that we read off an H-R diagram. Nonetheless, spectroscopic parallax is often the only means that an astronomer has to estimate the distance to remote stars.

What has been left out of this discussion is why different stars have different spectral types and luminosities. One key factor, as we shall see, turns out to be the mass of the star.

**17-9 Observing binary star systems reveals the masses of stars**

We now know something about the sizes, temperatures, and luminosities of stars. To complete our picture of the physical properties of stars, we need to know their masses. In this section, we will see that stars come in a wide range of masses. We will also discover an important relationship between the mass and luminosity of main-sequence stars. This relationship is crucial to understanding why some main-sequence stars are hot and luminous, while others are cool and dim. It will also help us understand what happens to a star as it ages and evolves.

Determining the masses of stars is not trivial, however. The problem is that there is no practical, direct way to measure the mass of an isolated star. Fortunately for astronomers, about half of the visible stars in the night sky are not isolated individuals. Instead, they are multiple-star systems, in which two or more stars orbit each other. By carefully observing the motions of these stars, astronomers can glean important information about their masses.

**Binary Stars**

A pair of stars located at nearly the same position in the night sky is called a double star. The Anglo-German astronomer William Herschel made the first organized search for such pairs. Between 1782 and 1821, he published three catalogs listing more than 800 double stars. Late in the nineteenth century, his son, John Herschel, discovered 10,000 more doubles. Some of these double stars are optical double stars, which are two stars that lie along nearly the same line of sight but are actually at very different distances from us. But many double stars are true binary stars, or
binaries—pairs of stars that actually orbit each other. Figure 17-19 shows an example of this orbital motion.

When astronomers can actually see the two stars orbiting each other, a binary is called a visual binary. By observing the binary over an extended period, astronomers can plot the orbit that one star appears to describe around the other, as shown in the center diagram in Figure 17-19.

In fact, both stars in a binary system are in motion: They orbit each other because of their mutual gravitational attraction, and their orbital motions obey Kepler’s third law as formulated by Isaac Newton (see Section 4-7 and Box 4-4). This law can be written as follows:

Kepler’s third law for binary star systems

\[ M_1 + M_2 = \frac{a^3}{P^2} \]

Here \( a \) is the semimajor axis of the elliptical orbit that one star appears to describe around the other, plotted as in the center diagram in Figure 17-19. As this equation indicates, if we can measure this semimajor axis \( a \) and the orbital period \( P \), we can learn something about the masses of the two stars.

In principle, the orbital period of a visual binary is easy to determine. All you have to do is see how long it takes for the two stars to revolve once about each other. The two stars shown in Figure 17-19 are relatively close, about 2.5 AU on average, and their orbital period is only 10 years. Many binary systems have...
much larger separations, however, and the period may be so long that more than one astronomer’s lifetime is needed to complete the observations.

Determining the semimajor axis of an orbit can also be a challenge. The angular separation between the stars can be determined by observation. To convert this angle into a physical distance between the stars, we need to know the distance between the binary and the Earth. This can be found from parallax measurements or by using spectroscopic parallax. The astronomer must also take into account how the orbit is tilted to our line of sight.

Once both $P$ and $a$ have been determined, Kepler’s third law can be used to calculate $M_1 + M_2$, the sum of the masses of the two stars in the binary system. But this analysis tells us nothing about the individual masses of the two stars. To obtain these, more information about the motions of the two stars is needed.

Each of the two stars in a binary system actually moves in an elliptical orbit about the center of mass of the system. Imagine two children sitting on opposite ends of a seesaw (Figure 17-20a). For the seesaw to balance properly, they must position themselves so that their center of mass—an imaginary point that lies along a line connecting their two bodies—is at the fulcrum, or pivot point of the seesaw. If the two children have the same mass, the center of mass lies midway between them, and they should sit equal distances from the fulcrum. If their masses are different, the center of mass is closer to the heavier child.

Just as the seesaw naturally balances at its center of mass, the two stars that make up a binary system naturally orbit around their center of mass (Figure 17-20b). The center of mass always lies along the line connecting the two stars and is closer to the more massive star.

The center of mass of a visual binary is located by plotting the separate orbits of the two stars, as in Figure 17-20b, using the background stars as reference points. The center of mass lies at the common focus of the two elliptical orbits. Comparing the relative sizes of the two orbits around the center of mass yields the ratio of the two stars’ masses, $M_1/M_2$. The sum $M_1 + M_2$ is already known from Kepler’s third law, so the individual masses of the two stars can then be determined.

**Main-Sequence Masses and the Mass-Luminosity Relation**

Years of careful, patient observations of binaries have slowly yielded the masses of many stars. As the data accumulated, an important trend began to emerge: For main-sequence stars, there is a direct correlation between mass and luminosity. The more massive a main-sequence star, the more luminous it is. Figure 17-21 depicts this mass-luminosity relation as a graph. The range of stellar masses extends from less than 0.1 of a solar mass to more than 50 solar masses. The Sun’s mass lies between these extremes.

The Cosmic Connections figure on the next page depicts the mass-luminosity relation for main-sequence stars on an H-R diagram. This figure shows the main sequence on an H-R diagram is a progression in mass as well as in luminosity and surface temperature. The hot, bright, bluish stars in the upper left corner of an H-R diagram are the most massive main-sequence stars. Likewise, the dim, cool, reddish stars in the lower right corner of an H-R diagram are the least massive. Main-sequence stars of intermediate temperature and luminosity also have intermediate masses.

The mass of a main-sequence star also helps determine its radius. Referring back to Figure 17-15b, we see that if we go along the main sequence from low luminosity to high luminosity, the radius of the star increases. Thus, we have the following general rule for main-sequence stars:

*The greater the mass of a main-sequence star, the greater its luminosity, its surface temperature, and its radius.*
The main sequence is an arrangement of stars according to their mass. The most massive main-sequence stars have the greatest luminosity, largest radius, and highest surface temperature. This is a consequence of the behavior of thermonuclear reactions at the core of a main-sequence star.

- A star with 60 solar masses has much higher pressure and temperature at its core than does the Sun.
- This causes thermonuclear reactions in the core to occur much more rapidly and release energy at a much faster rate — 790,000 times faster than in the Sun.
- Energy is emitted from the star’s surface at the same rate that it is released in the core, so the star has 790,000 times the Sun’s luminosity.
- The tremendous rate of energy release also heats the star’s interior tremendously, increasing the star’s internal pressure. This inflates the star to 15 times the Sun’s radius.
- The star’s surface must be at a high temperature (about 44,500 K) in order for it to emit energy into space at such a rapid rate.

- A star with 0.21 solar mass has much lower pressure and temperature at its core than does the Sun.
- This causes thermonuclear reactions in the core to occur much more slowly and release energy at a much slower rate — 0.011 times as fast as in the Sun.
- Energy is emitted from the star’s surface at the same rate that it is released in the core, so the star has 0.011 of the Sun’s luminosity.
- The low rate of energy release supplies relatively little heat to the star’s interior, so the star’s internal pressure is low. Hence the star’s radius is only 0.33 times the Sun’s radius.
- The star’s surface need be at only a low temperature (about 3200 K) to emit energy into space at such a relatively slow rate.
The Mass-Luminosity Relation For main-sequence stars, there is a direct correlation between mass and luminosity—the more massive a star, the more luminous it is. A main-sequence star of mass 10 \( M_\odot \) (that is, 10 times the Sun's mass) has roughly 3000 times the Sun's luminosity (3000 \( L_\odot \)); one with 0.1 \( M_\odot \) has a luminosity of only about 0.001 \( L_\odot \).

**Mass and Main-Sequence Stars**

Why is mass the controlling factor in determining the properties of a main-sequence star? The answer is that all main-sequence stars are objects like the Sun, with essentially the same chemical composition as the Sun but with different masses. Like the Sun, all main-sequence stars shine because thermonuclear reactions at their cores convert hydrogen to helium and release energy. The greater the total mass of the star, the greater the pressure and temperature at the core, the more rapidly thermonuclear reactions take place in the core, and the greater the energy output—that is, the luminosity—of the star. In other words, the greater the mass of a main-sequence star, the greater its luminosity. This statement is just the mass-luminosity relation, which we can now recognize as a natural consequence of the nature of main-sequence stars.

Like the Sun, main-sequence stars are in a state of both hydrostatic equilibrium and thermal equilibrium (see Section 16-2). Calculations using models of a main-sequence star's interior (like the solar models we discussed in Section 16-2) show that to maintain equilibrium, a more massive star must have a larger radius and a higher surface temperature. This is just what we see when we plot the curve of the main sequence on an H-R diagram (see Figure 17-15b). As you move up the main sequence from less massive stars (at the lower right in the H-R diagram) to more massive stars (at the upper left), the radius and surface temperature both increase.

Calculations using hydrostatic and thermal equilibrium also show that if a star's mass is less than about 0.08\( M_\odot \), the core pressure and temperature are too low for thermonuclear reactions to take place. The “star” is then a brown dwarf. Brown dwarfs also obey a mass-luminosity relation: The greater the mass, the faster the brown dwarf contracts because of its own gravity, the more rapidly it radiates energy into space, and, hence, the more luminous the brown dwarf is.

**CAUTION!** The mass-luminosity relation we have discussed applies to main-sequence stars only. There are no simple mass-luminosity relations for giant, supergiant, or white dwarf stars. Why these stars lie where they do on an H-R diagram will become apparent when we study the evolution of stars in Chapters 19 and 20. We will find that main-sequence stars evolve into giant and supergiant stars, and that some of these eventually end their lives as white dwarfs.

**17-10 Spectroscopy makes it possible to study binary systems in which the two stars are close together**

We have described how the masses of stars can be determined from observations of visual binaries, in which the two stars can be distinguished from each other. But if the two stars in a binary system are too close together, the images of the two stars can blend to produce the semblance of a single star. Happily, in many cases we can use spectroscopy to decide whether a seemingly single star is in fact a binary system. Spectroscopic observations of binaries provide additional useful information about star masses.

Some binaries are discovered when the spectrum of a star shows incongruous spectral lines. For example, the spectrum of what appears to be a single star may include both strong hydrogen lines (characteristic of a type A star) and strong absorption bands of titanium oxide (typical of a type M star). Because a single star cannot have the differing physical properties of these two spectral types, such a star must actually be a binary system that is too far away for us to resolve its individual stars. A binary system detected in this way is called a spectrum binary.

Other binary systems can be detected using the Doppler effect. If a star is moving toward the Earth, its spectral lines are displaced toward the short-wavelength (blue) end of the spectrum. Conversely, the spectral lines of a star moving away from us are shifted toward the long-wavelength (red) end of the spectrum. The upper portion of Figure 17-22 applies these ideas to a hypothetical binary star system with an orbital plane that is edge-on to our line of sight.

As the two stars move around their orbits, they periodically approach and recede from us. Hence, the spectral lines of the two stars are alternately blueshifted and redshifted. The two stars in this hypothetical system are so close together that they appear through a telescope as a single star with a single spectrum.
Because one star shows a blueshift while the other is showing a redshift, the spectral lines of the binary system appear to split apart and rejoin periodically. Stars whose binary character is revealed by such shifting spectral lines are called spectroscopic binaries.

Exploring Spectroscopic Binary Stars

To analyze a spectroscopic binary, astronomers measure the wavelength shift of each star’s spectral lines and use the Doppler shift formula (introduced in Section 5-9 and Box 5-6) to determine the radial velocity of each star—that is, how fast and in what direction it is moving along our line of sight. The lower portion of Figure 17-22 shows a graph of the radial velocity versus time, called a radial velocity curve, for the binary system HD 171978. Each of the two stars alternately approaches and recedes as it orbits around the center of mass. The pattern of the curves repeats every 15 days, which is the orbital period of the binary.

Figure 17-23 shows two spectra of the spectroscopic binary \( \kappa \) (kappa) Arietis taken a few days apart. In Figure 17-23a, two sets of spectral lines are visible, offset slightly in opposite directions from the normal positions of these lines. This corresponds to stage 1 or stage 3 in Figure 17-22; one of the orbiting stars is moving toward the Earth and has its spectral lines blueshifted, and the other star is moving away from the Earth and has its lines redshifted. A few days later, the stars have progressed along their orbits so that neither star is moving toward or away from the Earth, corresponding to stage 2 or stage 4 in Figure 17-22. At this time there are no Doppler shifts, and the spectral lines of both stars are at the same positions. That is why only one set of spectral lines appears in Figure 17-23b.

It is important to emphasize that the Doppler effect applies only to motion along the line of sight. Motion perpendicular to
When one of the stars in a spectroscopic binary is moving toward us and the other is receding from us, we see two sets of spectral lines due to the Doppler shift.

When both stars are moving perpendicular to our line of sight, there is no Doppler splitting and we see a single set of spectral lines.

**Figure 17-23**

A Spectroscopic Binary The visible-light spectrum of the double-line spectroscopic binary \( \kappa \) (kappa) Arietis has spectral lines that shift back and forth as the two stars revolve about each other. (Lick Observatory)

the line of sight does not affect the observed wavelengths of spectral lines. Hence, the ideal orientation for a spectroscopic binary is to have the stars orbit in a plane that is edge-on to our line of sight. (By contrast, a visual binary is best observed if the orbital plane is face-on to our line of sight.) For the Doppler shifts to be noticeable, the orbital speeds of the two stars should be at least a few kilometers per second.

The binaries depicted in Figures 17-22 and 17-23 are called double-line spectroscopic binaries, because the spectral lines of both stars in the binary system can be seen. Most spectroscopic binaries, however, are single-line spectroscopic binaries: One of the stars is so dim that its spectral lines cannot be detected. The star is obviously a binary, however, because its spectral lines shift back and forth, thereby revealing the orbital motions of two stars about their center of mass.

As for visual binaries, spectroscopic binaries allow astronomers to learn about stellar masses. From a radial velocity curve, one can find the ratio of the masses of the two stars in a binary. The sum of the masses is related to the orbital speeds of the two stars by Kepler’s laws and Newtonian mechanics. If both the ratio of the masses and their sum are known, the individual masses can be determined using algebra. However, determining the sum of the masses requires that we know how the binary orbits are tilted from our line of sight. This is because the Doppler shifts reveal only the radial velocities of the stars rather than their true orbital speeds. This tilt is often impossible to determine, because we cannot see the individual stars in the binary. Thus, the masses of stars in spectroscopic binaries tend to be uncertain.

There is one important case in which we can determine the orbital tilt of a spectroscopic binary. If the two stars are observed to eclipse each other periodically, then we must be viewing the orbit nearly edge-on. As we will see next, individual stellar masses—as well as other useful data—can be determined if a spectroscopic binary also happens to be such an eclipsing binary.

### 17-11 Light curves of eclipsing binaries provide detailed information about the two stars

Some binary systems are oriented so that the two stars periodically eclipse each other as seen from the Earth. These eclipsing binaries can be detected even when the two stars cannot be resolved visually as two distinct images in the telescope. The apparent brightness of the image of the binary dims briefly each time one star blocks the light from the other.

Using a sensitive detector at the focus of a telescope, an astronomer can measure the incoming light intensity quite accurately and create a light curve (Figure 17-24). The shape of the light curve for an eclipsing binary reveals at a glance whether the eclipse is partial or total (compare Figures 17-24a and 17-24b). Figure 17-24d shows an observation of a binary system undergoing a total eclipse.

In fact, the light curve of an eclipsing binary can yield a surprising amount of information. For example, the ratio of the surface temperatures can be determined from how much their combined light is diminished when the stars eclipse each other. Also, the duration of a mutual eclipse tells astronomers about the relative sizes of the stars and their orbits.

If the eclipsing binary is also a double-line spectroscopic binary, an astronomer can calculate the mass and radius of each star from the light curves and the velocity curves. Unfortunately, very few binary stars are of this ideal type. Stellar radii determined in this way agree well with the values found using the Stefan-Boltzmann law, as described in Section 17-6.

The shape of a light curve can reveal many additional details about a binary system. In some binaries, for example, the gravitational pull of one star distorts the other, much as the Moon distorts the Earth’s oceans in producing tides (see Figure 4-24). Figure 17-24c shows how such tidal distortion gives the light curve a different shape than in Figure 17-24b.

Information about stellar atmospheres can also be derived from light curves. Suppose that one star of a binary is a luminous main-sequence star and the other is a bloated red giant. By observing exactly how the light from the bright main-sequence star is gradually cut off as it moves behind the edge of the red giant during the beginning of an eclipse, astronomers can infer the pressure and density in the upper atmosphere of the red giant.

Binary systems are tremendously important because they enable astronomers to measure stellar masses as well as other key properties of stars. In the next several chapters, we will use this information to help us piece together the story of stellar evolution—how stars are born, evolve, and eventually die.
The Nature of the Stars

Key Words

Terms preceded by an asterisk (*) are discussed in the Boxes.

- absolute magnitude, p. xx
- apparent brightness (brightness), p. xx
- apparent magnitude, p. xx
- binary star (binary), p. xx
- brown dwarf, p. xx
- center of mass, p. xx
- color ratio, p. xx
- distance modulus, p. xx
- double star, p. xx
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Key Ideas

Measuring Distances to Nearby Stars: Distances to the nearer stars can be determined by parallax, the apparent shift of a star against the background stars observed as the Earth moves along its orbit.

- Parallax measurements made from orbit, above the blurring effects of the atmosphere, are much more accurate than those made with Earth-based telescopes.
- Stellar parallaxes can only be measured for stars within a few hundred parsecs.

The Inverse-Square Law: A star’s luminosity (total light output), apparent brightness, and distance from the Earth are related by
the inverse-square law. If any two of these quantities are known, the third can be calculated.

**The Population of Stars:** Stars of relatively low luminosity are more common than more luminous stars. Our own Sun is a rather average star of intermediate luminosity.

**The Magnitude Scale:** The apparent magnitude scale is an alternative way to measure a star’s apparent brightness.

- The absolute magnitude of a star is the apparent magnitude it would have if viewed from a distance of 10 parsecs. A version of the inverse-square law relates a star’s absolute magnitude, apparent magnitude, and distance.

**Photometry and Color Ratios:** Photometry measures the apparent brightness of a star. The color ratios of a star are the ratios of brightness values obtained through different standard filters, such as the U, B, and V filters. These ratios are a measure of the star’s surface temperature.

**Spectral Types:** Stars are classified into spectral types (subdivisions of the spectral classes O, B, A, F, G, K, and M), based on the major patterns of spectral lines in their spectra. The spectral class and type of a star is directly related to its surface temperature: O stars are the hottest and M stars are the coolest.

- Most brown dwarfs are in even cooler spectral classes called L and T. Unlike true stars, brown dwarfs are too small to sustain thermonuclear fusion.

**Hertzsprung-Russell Diagram:** The Hertzsprung-Russell (H-R) diagram is a graph plotting the absolute magnitudes of stars against their spectral types—or, equivalently, their luminosities against surface temperatures.

- The positions on the H-R diagram of most stars are along the main sequence, a band that extends from high luminosity and high surface temperature to low luminosity and low surface temperature.
- On the H-R diagram, giant and supergiant stars lie above the main sequence, while white dwarfs are below the main sequence.
- By carefully examining a star’s spectral lines, astronomers can determine whether that star is a main-sequence star, giant, supergiant, or white dwarf. Using the H-R diagram and the inverse-square law, the star’s luminosity and distance can be found without measuring its stellar parallax.

**Binary Stars:** Binary stars, in which two stars are held in orbit around each other by their mutual gravitational attraction, are surprisingly common. Those that can be resolved into two distinct star images by an Earth-based telescope are called visual binaries.

- Each of the two stars in a binary system moves in an elliptical orbit about the center of mass of the system.
- Binary stars are important because they allow astronomers to determine the masses of the two stars in a binary system. The masses can be computed from measurements of the orbital period and orbital dimensions of the system.

**Mass-Luminosity Relation for Main-Sequence Stars:** Main-sequence stars are stars like the Sun but with different masses.

- The mass-luminosity relation expresses a direct correlation between mass and luminosity for main-sequence stars. The greater the mass of a main-sequence star, the greater its luminosity (and also the greater its radius and surface temperature).

**Spectroscopic Observations of Binary Stars:** Some binaries can be detected and analyzed, even though the system may be so distant or the two stars so close together that the two star images cannot be resolved.

- A spectrum binary appears to be a single star but has a spectrum with the absorption lines for two distinctly different spectral types.
- A spectroscopic binary has spectral lines that shift back and forth in wavelength. This is caused by the Doppler effect, as the orbits of the stars carry them first toward then away from the Earth.
- An eclipsing binary is a system whose orbits are viewed nearly edge-on from the Earth, so that one star periodically eclipses the other. Detailed information about the stars in an eclipsing binary can be obtained from a study of the binary’s radial velocity curve and its light curve.

### Questions

#### Review Questions

1. Explain the difference between a star’s apparent brightness and its luminosity.
2. Describe how the parallax method of finding a star’s distance is similar to binocular (two-eye) vision in humans.
3. Why does it take at least six months to make a measurement of a star’s parallax?
4. Why are measurements of stellar parallax difficult to make? What are the advantages of making these measurements from orbit?
5. What is the inverse-square law? Use it to explain why an ordinary lightbulb can appear brighter than a star, even though the lightbulb emits far less light energy per second.
6. Briefly describe how you would determine the luminosity of a nearby star. Of what value is knowing the luminosity of various stars?
7. Which are more common, stars more luminous than the Sun or stars less luminous than the Sun?
8. Why is the magnitude scale called a “backward” scale? What is the difference between apparent magnitude and absolute magnitude?
9. The star Zubenelgenubi (from the Arabic for “scorpion’s southern claw”) has apparent magnitude +2.75, while the star Sulafat (Arabic for “tortoise”) has apparent magnitude +3.25. Which star appears brighter? From this information alone, what can you conclude about the luminosities of these stars? Explain.
10. Explain why the color ratios of a star are related to the star’s surface temperature.
11. Would it be possible for a star to appear bright when viewed through a U filter or a V filter, but dim when viewed through a B filter? Explain.
12. Which gives a more accurate measure of a star’s surface temperature, its color ratios or its spectral lines? Explain.
13. Menkalinan (Arabic for “shoulder of the rein-holder”) is an A2 star in the constellation Auriga (the Charioteer). What is its spectral class? What is its spectral type? Which gives a more precise description of the spectrum of Menkalinan?
14. What are the most prominent absorption lines you would expect to find in the spectrum of a star with a surface temperature of (a) 35,000 K, (b) 2800 K, and (c) 5800 K (like the Sun)? Briefly describe why these stars have such different spectra even though they have essentially the same chemical composition.
15. A fellow student expresses the opinion that since the Sun’s spectrum has only weak absorption lines of hydrogen, this element cannot be a major constituent of the Sun. How would you enlighten this person?
16. If a red star and a blue star both have the same radius and both are the same distance from the Earth, which one looks brighter in the night sky? Explain why.
17. If a red star and a blue star both appear equally bright and both are the same distance from the Earth, which one has the larger radius? Explain why.
18. If a red star and a blue star both have the same radius and both appear equally bright, which one is farther from Earth? Explain why.
19. Sketch a Hertzsprung-Russell diagram. Indicate the regions on your diagram occupied by (a) main-sequence stars, (b) red giants, (c) supergiants, (d) white dwarfs, and (e) the Sun.
20. Most of the bright stars in the night sky (see Appendix 5) are giants and supergiants. How can this be, if giants and supergiants make up only 1% of the population of stars?
21. Explain why the dashed lines in Figure 17-15b slope down and to the right.
22. Some giant and supergiant stars are of the same spectral type (G2) as the Sun. What aspects of the spectrum of a G2 star would you concentrate on to determine the star’s luminosity class? Explain what you would look for.
23. Briefly describe how you would determine the distance to a star whose parallax is too small to measure.
24. What information about stars do astronomers learn from binary systems that cannot be learned in any other way? What measurements do they make of binary systems to garner this information?
25. Suppose that you want to determine the temperature, diameter, and luminosity of an isolated star (not a member of a binary system). Which of these physical quantities require you to know the distance to the star? Explain.
26. What is the mass-luminosity relation? Does it apply to stars of all kinds?
27. Use Figure 17-21 to (a) estimate the mass of a main-sequence star that is 1000 times as luminous as the Sun, and (b) estimate the luminosity of a main-sequence star whose mass is one-fifth that of the Sun. Explain your answers.
28. Which is more massive, a red main-sequence star or a blue main-sequence star? Which has the greater radius? Explain your answers.
29. How do white dwarfs differ from brown dwarfs? Which are more massive? Which are larger in radius? Which are denser?
30. Sketch the radial velocity curves of a binary consisting of two identical stars moving in circular orbits that are (a) perpendicular to and (b) parallel to our line of sight.
31. Give two reasons why a visual binary star is unlikely to also be a spectroscopic binary star.
32. Sketch the light curve of an eclipsing binary consisting of two identical stars in highly elongated orbits oriented so that (a) their major axes are pointed toward the Earth and (b) their major axes are perpendicular to our line of sight.

Advanced Questions

Questions preceded by an asterisk (*) involve topics discussed in the Boxes.

Problem-solving tips and tools

Look carefully at the worked examples in Boxes 17-1, 17-2, 17-3, and 17-4 before attempting these exercises. For data on the planets, see Table 7-1 or Appendices 1 and 2 at the back of this book. Remember that a telescope’s light-gathering power is proportional to the area of its objective or primary mirror. The volume of a sphere of radius r is $4\pi r^3/3$. Make use of the H-R diagrams in this chapter to answer questions involving spectroscopic parallax. As Box 17-3 shows, some of the problems concerning magnitudes may require facility with logarithms.

33. Find the average distance from the Sun to Neptune in parsecs. Compared to Neptune, how many times farther away from the Sun is Proxima Centauri?
34. Suppose that a dim star were located 2 million AU from the Sun. Find (a) the distance to the star in parsecs and (b) the parallax angle of the star. Would this angle be measurable with present-day techniques?
35. The star GJ 1156 has a parallax angle of 0.153 arcsec. How far away is the star?
36. Kapteyn’s star (named after the Dutch astronomer who found it) has a parallax of 0.255 arcsec, a proper motion of 8.67 arcsec per year, and a radial velocity of +246 km/s. (a) What is the star’s tangential velocity? (b) What is the star’s actual speed relative to the Sun? (c) Is Kapteyn’s star moving toward the Sun or away from the Sun? Explain.
37. How far away is a star that has a proper motion of 0.08 arcseconds per year and a tangential velocity of 40 km/s? For a star at this distance, what would its tangential velocity have to be in order for it to exhibit the same proper motion as Barnard’s star (see Box 17-1)?
38. The space velocity of a certain star is 120 km/s and its radial velocity is 72 km/s. Find the star’s tangential velocity.
39. In the spectrum of a particular star, the Balmer line Hα has a wavelength of 656.15 nm. The laboratory value for the wavelength of Hα is 656.28 nm. (a) Find the star’s radial velocity. (b) Is this star approaching us or moving away? Explain. (c) Find the wavelength at which you would expect to find Hα in the spectrum of this star, given that the laboratory wavelength of Hα is 486.13 nm. (d) Do your answers depend on the distance from the Sun to this star? Why or why not?
40. Derive the equation given in Box 17-1 relating proper motion and tangential velocity. (Hint: See Box 1-1.)

41. How much dimmer does the Sun appear from Neptune than from Earth? (Hint: The average distance between a planet and the Sun equals the semimajor axis of the planet’s orbit.)

42. Stars A and B are both equally bright as seen from Earth, but A is 120 pc away while B is 24 pc away. Which star has the greater luminosity? How many times greater is it?

43. Stars C and D both have the same luminosity, but C is 32 pc from Earth while D is 128 pc from Earth. Which star appears brighter as seen from Earth? How many times brighter is it?

44. Suppose two stars have the same apparent brightness, but one star is 8 times farther away than the other. What is the ratio of their luminosities? Which one is more luminous, the closer star or the farther star?

45. The solar constant, equal to 1370 W/m², is the amount of light energy from the Sun that falls on 1 square meter of the Earth’s surface in 1 second (see Section 17-2). What would the distance between the Earth and the Sun have to be in order for the solar constant to be 1 watt per square meter (1 W/m²)?

46. The star Procyon in Canis Minor (the Small Dog) is a prominent star in the winter sky, with an apparent brightness 1.3 × 10⁻¹¹ that of the Sun. It is also one of the nearest stars, being only 3.50 parsecs from Earth. What is the luminosity of Procyon? Express your answer as a multiple of the Sun’s luminosity.

47. The star HIP 92403 (also called Ross 154) is only 2.97 parsecs from Earth but can be seen only with a telescope, because it is 60 times dimmer than the dimmest star visible to the unaided eye. How close to us would this star have to be in order for it to be visible without a telescope? Give your answer in parsecs and in AU. Compare with the semimajor axis of Pluto’s orbit around the Sun.

48. The star HIP 72509 has an apparent magnitude of +12.1 and a parallax angle of 0.222 arcsecond. (a) Determine its absolute magnitude. (b) Find the approximate ratio of the luminosity of HIP 72509 to the Sun’s luminosity.

49. Suppose you can just barely see a twelfth-magnitude star through an amateur’s 6-inch telescope. What is the magnitude of the dimmest star you could see through a 60-inch telescope?

50. A certain type of variable star is known to have an average absolute magnitude of 0.0. Such stars are observed in a particular star cluster to have an average apparent magnitude of +14.0. What is the distance to that star cluster?

51. (a) Find the absolute magnitudes of the brightest and dimmest of the labeled stars in Figure 17-6b. Assume that all of these stars are 110 pc from Earth. (b) If a star in the Pleiades cluster is just bright enough to be seen from Earth with the naked eye, what is its absolute magnitude? Is such a star more or less luminous than the Sun? Explain.

52. (a) On a copy of Figure 17-8, sketch the intensity curve for a blackbody at a temperature of 3000 K. Note that this figure shows a smaller wavelength range than Figure 17-7a. (b) Repeat part (a) for a blackbody at 12,000 K (see Figure 17-7c). (c) Use your sketches from parts (a) and (b) to explain why the color ratios \( b_V/b_B \) and \( b_B/b_V \) are less than 1 for very hot stars but greater than 1 for very cool stars.

53. Astronomers usually express a star’s color using apparent magnitudes. The star’s apparent magnitude as viewed through a B filter is called \( m_B \), and its apparent magnitude as viewed through a V filter is \( m_V \). The difference \( m_B - m_V \) is called the B–V color index (“B minus V”). Is the B–V color index positive or negative for very hot stars? What about very cool stars? Explain your answers.

54. (See Question 53.) The B–V color index is related to the color ratio \( b_V/b_B \) by the equation

\[
 m_B - m_V = 2.5 \log \left( \frac{b_V}{b_B} \right)
\]

(a) Explain why this equation is correct. (b) Use the data in Table 17-1 to calculate the B–V color indices for Bellatrix, the Sun, and Betelgeuse. From your results, describe a simple rule that relates the value of the B–V color index to a star’s color.

55. The bright star Rigel in the constellation Orion has a surface temperature about 1.6 times that of the Sun. Its luminosity is about 64,000 L☉. What is Rigel’s radius compared to the radius of the Sun?

56. (See Figure 17-12.) What temperature and spectral classification would you give to a star with equal line strengths of hydrogen (H) and neutral helium (He I)? Explain.

57. The Sun’s surface temperature is 5800 K. Using Figure 17-12, arrange the following absorption lines in the Sun’s spectrum from the strongest to the weakest, and explain your reasoning: (i) neutral calcium; (ii) singly ionized calcium; (iii) neutral iron; (iv) singly ionized iron.

58. Star P has one-half the radius of star Q. Stars P and Q have surface temperatures 4000 K and 8000 K, respectively. Which star has the greater luminosity? How many times greater is it?

59. Star X has 12 times the luminosity of star Y. Stars X and Y have surface temperatures 3500 K and 7800 K, respectively. Which star has the larger radius? How many times larger is it?

60. Suppose a star experiences an outburst in which its surface temperature doubles but its average density (its mass divided by its volume) decreases by a factor of 8. The mass of the star stays the same. By what factors do the star’s radius and luminosity change?

61. The Sun experiences solar flares (see Section 16-10). The amount of energy radiated by even the strongest solar flare is not enough to have an appreciable effect on the Sun’s luminosity. But when a flare of the same size occurs on a main-sequence star of spectral class M, the star’s brightness can increase by as much as a factor of 2. Why should there be an appreciable increase in brightness for a main-sequence M star but not for the Sun?

62. The bright star Zubeneschmali (β Librae) is of spectral type B8 and has a luminosity of 130 L☉. What is the star’s approximate surface temperature? How does its radius compare to that of the Sun?
63. Castor (α Geminorum) is an A1 V star with an apparent brightness of \(4.4 \times 10^{-12}\) that of the Sun. Determine the approximate distance from the Earth to Castor (in parsecs).

64. A brown dwarf called CoD–33°0779 B has a luminosity of 0.0025L⊙. It has a relatively high surface temperature of 2550 K, which suggests that it is very young and has not yet had time to cool down by emitting radiation. (a) What is this brown dwarf’s spectral class? (b) Find the radius of CoD–33°0779 B. Express your answer in terms of the Sun’s radius and in kilometers. How does this compare to the radius of Jupiter? Is the name “dwarf” justified?

65. The star HD 3651 shown in Figure 17-13 has a mass of 0.79 M⊙. Its brown dwarf companion, HD 3651B, has about 40 times the mass of Jupiter. The average distance between the two stars is about 480 AU. How long does it take the two stars to complete one orbit around each other?

66. The visual binary 70 Ophiuchi (see the accompanying figure) has a period of 87.7 years. The parallax of 70 Ophiuchi is 0.2 arcsec, and the apparent length of the semimajor axis as seen through a telescope is 4.5 arcsec. (a) What is the distance to 70 Ophiuchi in parsecs? (b) What is the actual length of the semimajor axis in AU? (c) What is the sum of the masses of the two stars? Give your answer in solar masses.

67. An astronomer observing a binary star finds that one of the stars orbits the other once every 5 years at a distance of 10 AU. (a) Find the sum of the masses of the two stars. (b) If the mass ratio of the system is \(M_1/M_2 = 0.25\), find the individual masses of the stars. Give your answers in terms of the mass of the Sun.

**Discussion Questions**

68. From its orbit around the Earth, the Hipparcos satellite could measure stellar parallax angles with acceptable accuracy only if the angles were larger than about 0.002 arcsec. Discuss the advantages or disadvantages of making parallax measurements from a satellite in a large solar orbit, say at the distance of Jupiter from the Sun. If this satellite can also measure parallax angles of 0.002 arcsec, what is the distance of the most remote stars that can be accurately determined? How much bigger a volume of space would be covered compared to the Earth-based observations? How many more stars would you expect to be contained in that volume?

69. As seen from the starship Enterprise in the Star Trek television series and movies, stars appear to move across the sky due to the starship’s motion. How fast would the Enterprise have to move in order for a star 1 pc away to appear to move 1° per second? (Hint: The speed of the star as seen from the Enterprise is the same as the speed of the Enterprise relative to the star.) How does this compare with the speed of light? Do you think the stars appear to move as seen from an orbiting space shuttle, which moves at about 8 km/s?

70. It is desirable to be able to measure the radial velocity of stars (using the Doppler effect) to an accuracy of 1 km/s or better. One complication is that radial velocities refer to the motion of the star relative to the Sun, while the observations are made using a telescope on the Earth. Is it important to take into account the motion of the Earth relative to the Sun? Is it important to take into account the Earth’s rotational motion? To answer this question, you will have to calculate the Earth’s orbital speed and the speed of a point on the Earth’s equator (the part of the Earth’s surface that moves at the greatest speed because of the planet’s rotation). If one or both of these effects are of importance, how do you suppose astronomers compensate for them?

**Web/eBook Questions**

71. Search the World Wide Web for information about Gaia, a European Space Agency (ESA) spacecraft planned to extend the work carried out by Hipparcos. When is the spacecraft planned to be launched? How will Gaia compare to Hipparcos? For how many more stars will it be able to measure parallaxes? What other types of research will it carry out?

72. Search the World Wide Web for recent discoveries about brown dwarfs. Are all brown dwarfs found orbiting normal stars, or are they also found orbiting other brown dwarfs? Are any found in isolation (that is, not part of a binary system)? The Sun experiences flares (see Section 16-10), as do other normal stars; is there any evidence that brown dwarfs also experience flares? If so, is there anything unusual about these flares?

73. **Distances to Stars Using Parallax.** Access the Active Integrated Media Module “Using Parallax to Determine Distance” in Chapter 17 of the Universe Web site or eBook. Use this to determine the distance in parsecs and in light-years to each of the following stars: (a) Betelgeuse (parallax \(p = 0.00763\) arcsecond); (b) Vega (\(p = 0.129\) arcsecond); (c) Antares (\(p = 0.00540\) arcsecond); (d) Sirius (\(p = 0.379\) arcsecond).

74. **Finding Absolute Magnitudes.** Access the Active Integrated Media Module “The Distance-Magnitude Relationship” in Chapter 17 of the Universe Web site or eBook. Use this to determine the absolute magnitudes of stars with the following properties: (a) apparent magnitude +6.3, distance = 125 pc; (b) apparent magnitude +11.4, distance = 48 pc; (c) apparent magnitude +9.8, distance = 70 pc. (d) Rank the three stars from parts (a), (b), and (c) in order from highest to lowest luminosity.
75. The accompanying table lists five well-known red stars. It includes their right ascension and declination (celestial coordinates described in Box 2-1), apparent magnitudes, and color ratios. As their apparent magnitudes indicate, all these stars are somewhat variable. Observe at least two of these stars both by eye and through a small telescope. Is the reddish color of the stars readily apparent, especially in contrast to neighboring stars? (The Jesuit priest and astronomer Angelo Secchi named Y Canum Venaticorum “La Superba,” and μ Cephei is often called William Herschel’s “Garnet Star.”)

76. The accompanying table of double stars includes vivid examples of contrasting star colors. The table lists the angular separation between the stars of each double. Observe at least four of these double stars through a telescope. Use the spectral types listed to estimate the difference in surface temperature of the stars in each pair you observe. Does the double with the greatest difference in temperature seem to present the greatest color contrast? From what you see through the telescope and on what you know about the H-R diagram, explain why all the cool stars (spectral types K and M) listed are probably giants or supergiants.

77. Observe the eclipsing binary Algol (β Persei), using nearby stars to judge its brightness during the course of an eclipse. Algol has an orbital period of 2.87 days, and, with the onset of primary eclipse, its apparent magnitude drops from 2.1 to 3.4. It remains this faint for about 2 hours. The entire eclipse, from start to finish, takes about 10 hours. Consult the “Celestial Calendar” section of the current issue of Sky & Telescope for the predicted dates and times of the minima of Algol. Note that the schedule is given in Universal Time (the same as Greenwich Mean Time), so you will have to convert the time to that of your own time zone. Algol is normally the second brightest star in the constellation of Perseus. Because of its position on the celestial sphere (R.A. = 3° 08.2′南, Decl. = 40° 57′北), Algol is readily visible from northern latitudes during the fall and winter months.

78. Use the Starry Night Enthusiast™ program to investigate the brightest stars. Click on Home to show the sky as seen from your location. Set the date to today’s date and the time to midnight (12:00:00 A.M.). In the View menu, open the Constellations pane and click on Boundaries and Labels. You will now see the boundaries of the constellations. Open the Info pane and expand the Position in Space and Other Data lists. (a) Scroll around the sky and identify at least five of the brighter stars (shown as larger dots) and click on them to reveal relevant data in the Info pane. Make a list of these stars and record Luminosity and Distance from Sun from the Info pane. Which stars did you select? In which constellation does each of these stars lie? Which of these stars are listed in Appendix 5? Of these, which is the most luminous? Which is the most distant? (b) Set the date to six months from today, and again set
the time to 12:00:00 A.M. Which of the stars that you selected in part (a) are visible? (You can use the Find pane to attempt to locate your selected stars.) Which are not? Explain why the passage of six months should make a difference.

79. Use the Starry Night Enthusiast™ program to examine the nearby stars. Click on Favourites > Stars > Local Neighborhood and Stop time. Select View > Feet to hide the spacesuit image. Center this view upon the Sun by opening the Find pane and double-clicking on Sun. You are now 16.41 light years from the Sun, looking at the labeled nearby stars. Increase current elevation to about 70,000 light-years using the button on the Control Panel below the Viewing Location box (an upward-pointing triangle) to see these nearby stars within the Milky Way Galaxy. You can rotate the galaxy by placing the mouse cursor over the image and holding down the Shift key while holding down the mouse button and moving the mouse. (On a two-button mouse, hold down the left mouse button). Decrease current elevation to a distance of about 100 light-years from the Sun to return to the solar neighborhood. Again, you can rotate this swarm of stars by holding down the Shift key while holding down the mouse button and moving the mouse.

Open the Info pane. If you click the mouse while the cursor is over a star, you will see the star’s apparent magnitude as seen from Earth in the Other Data layer and its distance from the Sun in the Position in Space layer of the Info pane. (a) Select at least 5 stars within 50 light-years of the Sun and note their names, apparent magnitudes, luminosities, and distances from the Sun in a list. Which of these stars would be visible from Earth with the naked eye from a dark location? Which are visible with the naked eye from a brightly lit city? (Hint: The naked eye can see stars as faint as apparent magnitude $m = +6$ from a dark location, but only as faint as $m = +4$ from an inner city.) (b) Increase current elevation once more to about 1000 light-years from Earth and locate at least 5 stars that are further than 500 light-years from the Sun, making a list of these stars, their names, apparent magnitudes, luminosities and distances from the Sun. Which of these stars are visible from Earth with the naked eye from a dark location? Are the naked-eye stars more likely to be giants or supergiants, or are they more likely to be main-sequence stars? Explain your answer.