Learning objectives

- Describe the use of stellar spectra.
- Work with the HR diagram, including representation of stellar evolution.
- Apply the mass–luminosity relation.
- Describe Cepheid variables and their use as standard candles.
- Describe the nature of the stars in the main regions of the HR diagram.
- Understand the limits on mass for white dwarfs and neutron stars.

D2 Stellar characteristics and stellar evolution

This section deals with the lives of stars on the main sequence and their evolution away from it. We will see how **stellar spectra** may be used to determine the chemical composition of stars, and will study an important diagram called the Hertzsprung–Russell (HR) diagram. We will follow the **stellar evolution** on the HR diagram and meet important classes of stars such as Cepheid variables, **white dwarfs** and **red giants**.

D2.1 Stellar spectra

The energy radiated by a star is in the form of electromagnetic radiation and is distributed over an infinite range of wavelengths. A star is assumed to radiate like a black body. Figure **D.9** shows black-body spectra at various temperatures.

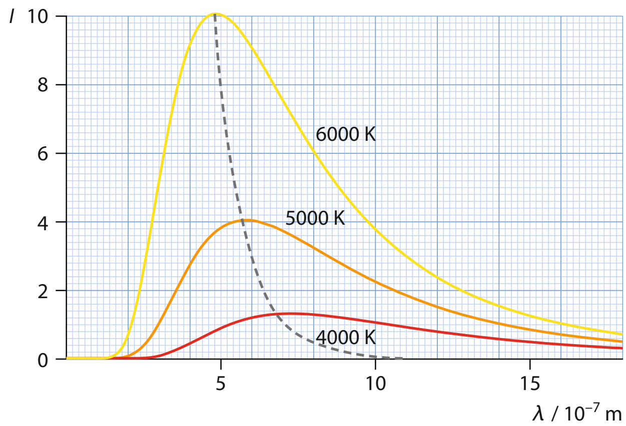


Figure D.9 Black-body radiation profiles at various temperatures. The broken lines show the variation with temperature of the peak intensity and the wavelength at which it occurs.

Much information can be determined about a star by examining its spectrum. The first piece of information is its surface temperature. Most of the energy is emitted around a wavelength called the peak wavelength. Calling this wavelength λ_0, we see that the colour of the star is mainly determined by the colour corresponding to λ_0.

The **Wien displacement law** relates the wavelength λ_0 to the surface temperature T:

$$\lambda_0 T = \text{constant} = 2.90 \times 10^{-3}\,\text{K m}$$

This implies that the higher the temperature, the lower the wavelength at which most of the energy is radiated.

Worked examples

D.9 The Sun has an approximate black-body spectrum with most of its energy radiated at a wavelength of 5.0×10^{-7} m. Find the surface temperature of the Sun.

From Wien's law, $5.0 \times 10^{-7}\,\text{m} \times T = 2.9 \times 10^{-3}\,\text{K m}$; that is, $T = 5800\,\text{K}$.

D.10 The Sun (radius $R = 7.0 \times 10^{8}$ m) radiates a total power of 3.9×10^{26} W. Find its surface temperature.

From $L = \sigma A T^4$ and $A = 4\pi R^2$, we find

$$T = \left(\frac{L}{\sigma 4\pi R^2}\right)^{1/4} \approx 5800\,\text{K}$$

The surface temperature of a star is determined by measuring the wavelength at which most of its radiation is emitted (see Figure **D.10**).

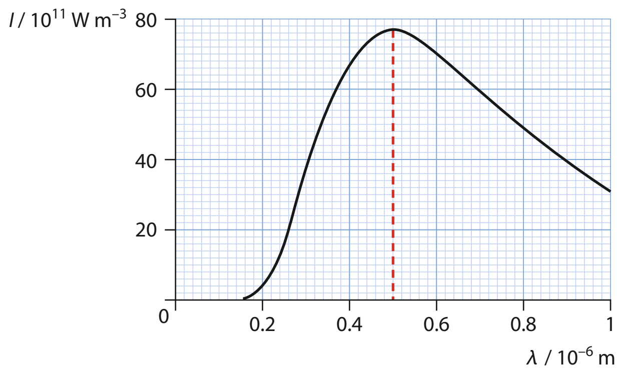


Figure D.10 The spectrum of this star shows a peak wavelength of 500 nm. Using Wien's law, we can determine its surface temperature.

The second important piece of information from a star's spectrum is its chemical composition. It is common to obtain an absorption spectrum in which dark lines are superimposed on a background of continuous colour (as in Figure **D.11**). Each dark line represents the absorption of light of a specific wavelength by a specific chemical element in the star's atmosphere.

Figure D.11 Absorption spectrum of a star, showing the absorption lines of hydrogen. A real spectrum would show very many dark lines corresponding to other elements as well.

It is known that most stars have essentially the same chemical composition, yet show different absorption spectra. The reason for this difference is that different stars have different temperatures. Consider two stars with the same content of hydrogen. One is hot, say at 25 000 K, and the other cool, say at 10 000 K. The hydrogen in the hot star is ionised, which means the electrons have left the hydrogen atoms. These atoms cannot absorb any light passing through them, since there are no bound electrons to absorb the photons and make transitions to higher-energy states. Thus, the hot star will not show any absorption lines at hydrogen wavelengths. The cooler star, however, has many of its hydrogen atoms in the energy state $n=2$. Electrons in this state can absorb photons to make transitions to states such as $n=3$ and $n=4$, giving rise to characteristic hydrogen absorption lines. Similarly, an even cooler star – of temperature, say, 3000 K – will have most of the electrons in its hydrogen atoms in the ground state, so they can only absorb photons corresponding to ultraviolet wavelengths. These will not result in dark lines in an optical spectrum.

Stars are divided into seven **spectral classes** according to their colour (see Table **D.3**). As we have seen, colour is related to surface temperature. The spectral classes are called O, B, A, F, G, K and M (remembered as Oh Be A Fine Girl/Guy Kiss Me!).

Spectral class	Colour	Temperature / K
O	Electric blue	25 000–50 000
B	Blue	12 000–25 000
A	White	7 500–12 000
F	Yellow–white	6 000–7 500
G	Yellow	4 500–6 000
K	Orange	3 000–4 500
M	Red	2 000–3 000

Table D.3 Colour and temperature characteristics of spectral classes.

It is known from spectral studies that hydrogen is the predominant element in normal **main-sequence** stars, making up upto 70% of their mass, followed by helium with at 28%; the rest is made up of heavier elements.

Spectral studies also give information on the star's velocity and rotation (through the Doppler shifting of spectral lines) and the star's magnetic field owing to the splitting of spectral lines in a magnetic field.

D2.2 The Hertzsprung–Russell diagram

Astronomers realised early on that there was a correlation between the luminosity of a star and its surface temperature. In the early part of the twentieth century, the Danish astronomer Ejnar Hertzsprung and the American astronomer Henry Norris Russell independently pioneered plots of stellar luminosities. Such plots are now called **Hertzsprung–Russell** (HR) **diagrams**. In the HR diagram in Figure **D.12**, the vertical axis represents luminosity in units of the Sun's luminosity (that is, 1 on the vertical axis corresponds to the solar luminosity, $L_{\odot}=3.9\times10^{26}$ W). The horizontal axis shows the surface temperature of the star (in kelvin). The temperature decreases to the right.

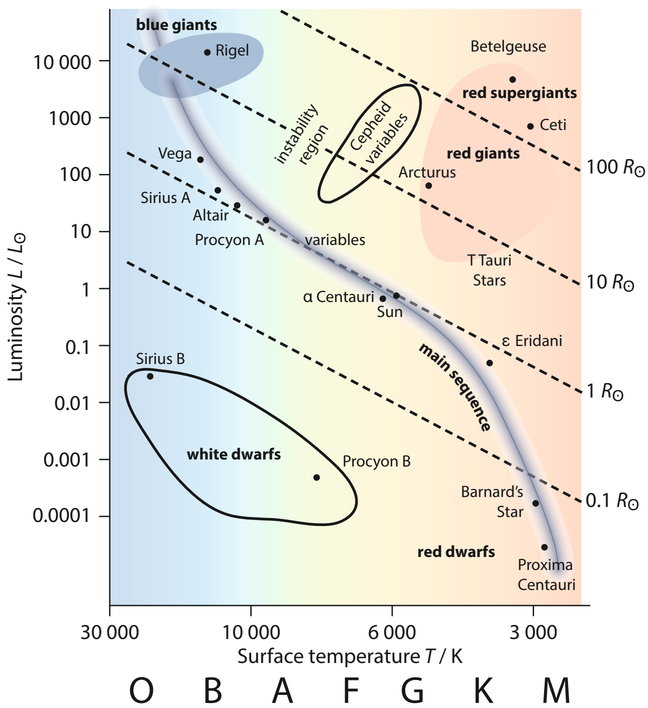


Figure D.12 A Hertzsprung–Russell diagram. Surface temperature increases to the left. Note that the scales are not linear.

Exam tip
It is very important that you clearly understand the HR diagram.

Also shown is the spectral class, which is an alternative way to label the horizontal axis. The luminosity in this diagram varies from 10^{-4} to 10^{4}, a full eight orders of magnitude, whereas the temperature varies from 3000 K to 30 000 K. For this reason, the scales on each axis are not linear.

The slanted dotted lines represent stars with the same radius, so our Sun and Procyon have about the same radius. The symbol $R_{\odot}$ stands for the radius of the Sun.

As more and more stars were placed on the HR diagram, it became clear that a pattern was emerging. The stars were not randomly distributed on the diagram. Three clear features emerge:

- Most stars fall on a strip extending diagonally across the diagram from top left to bottom right. This is called the **main sequence**.
- Some large stars, reddish in colour, occupy the top right. These are the **red giants** (large and cool). Above these are the **red supergiants** (very large and cool).
- The bottom left is a region of small stars known as **white dwarfs** (small and hot).

As we will see in Section **D2.3**, the higher the luminosity of a main-sequence star, the higher its mass. So as we move along the main sequence towards hotter stars, the masses of the stars increase. Thus, the right end of the main sequence is occupied by **red dwarfs** and the left by **blue giants**.

Note that, once we know the temperature of a star (for example, through its spectrum), the HR diagram can tell us its luminosity with an acceptable degree of accuracy, provided it is a main-sequence star.

Exam tip
Main-sequence stars: fuse hydrogen to form helium,
Red giants: bright, large, cool, reddish, tenuous.
Red supergiants: even larger and brighter than red giants,
White dwarfs: dim, small, hot, whitish, dense.

D2.3 Main-sequence stars

Our Sun is a typical member of the main sequence. It has a mass of 2×10^{30} kg, a radius of 7×10^{8} m and an average density of $1.4 \times 10^{3}\,\mathrm{kg\,m^{-3}}$, and it radiates at a rate of 3.9×10^{26} W. What distinguishes different main-sequence stars is their mass (see Figure **D.13**). Main-sequence stars produce enough energy in their core, from the nuclear fusion of hydrogen into helium, to exactly counterbalance the tendency of the star to collapse under its own weight. The common characteristic of all main-sequence stars is the fusion of hydrogen into helium.

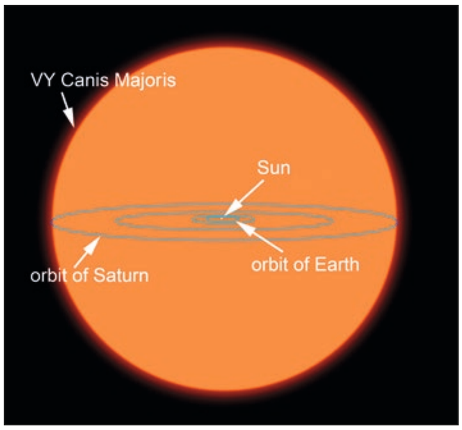


Figure D.13 Artist's impression of a comparison between our Sun and the red supergiant VY Canis Majoris.

D2.4 Red giants and red supergiants

Red giants are very large, cool stars with a reddish appearance. The luminosity of red giants is considerably greater than that of main-sequence stars of the same temperature. Treating them as black bodies radiating according to the Stefan–Boltzmann law means that a luminosity which is 10^{3} times greater than that of our Sun corresponds to a surface area which is 10^{3} times that of the Sun, and thus a radius about 30 times greater. The mass of a red giant can be as much as 100 times the mass of our Sun, but their huge size also implies small densities. A red giant will have a central hot core surrounded by an enormous envelope of extremely tenuous gas.

Red supergiants are even larger. Extreme examples include stars with radii that are 1500 times that of our Sun and luminosities of 5×10^{5} solar luminosities.

D2.5 White dwarfs

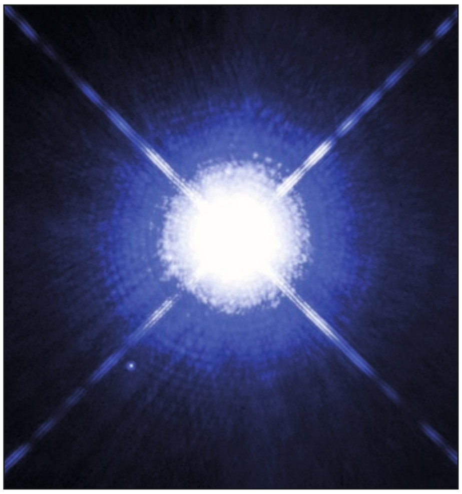


Figure D.14 Sirius A is the bright star in the middle of the photograph. Its white dwarf companion, Sirius B, is the tiny speck of light at the lower left. The rings and spikes are artefacts of the telescope's imaging system. The photograph has been overexposed so that the faint Sirius B can be seen.

White dwarf stars are common but their faintness makes them hard to detect. A well-known white dwarf is Sirius B, the second star in a binary star system (double star) whose other member, Sirius A, is the brightest star in the evening sky (Figure **D.14**).

Sirius A and Sirius B have about the same surface temperature (about 10 000 K) but the luminosity of Sirius B is about 10 000 times smaller. This means that it has a radius that is 100 times smaller than that of Sirius A. Here is a star with a mass roughly that of the Sun with a size similar to that of the Earth. This means that its density is about 10^6 times the density of the Earth!

Worked example

D.11 A main-sequence star emits most of its energy at a wavelength of 2.4×10^{-7} m. Its apparent brightness is measured to be $4.3 \times 10^{-9}\,\mathrm{W\,m^{-2}}$. Estimate the distance of the star.

From Wien's law, we find the temperature of the star to be given by

$$\lambda_0 T = 2.9 \times 10^{-3}\,\mathrm{K\,m}$$

$$\Rightarrow T = \frac{2.9 \times 10^{-3}}{2.4 \times 10^{-7}}\,\mathrm{K}$$

$$= 12\,000\,\mathrm{K}$$

From the HR diagram in Figure **D.12**, we see that such a temperature corresponds to a luminosity about 100 times that of the Sun: that is, $L = 3.9 \times 10^{28}$ W. Thus,

$$d = \sqrt{\frac{L}{4\pi b}} = \sqrt{\frac{3.9 \times 10^{28}}{4\pi \times 4.3 \times 10^{-9}}}\,\mathrm{m}$$

$$= 8.5 \times 10^{17}\,\mathrm{m} \approx 90\,\mathrm{ly} \approx 28\,\mathrm{pc}$$

D2.6 The mass–luminosity relation

Exam tip
The mass–luminosity relation can only be used for main-sequence stars.

For stars on the main sequence, there exists a relation between the mass and the luminosity of the star. The **mass–luminosity relation** states that

$$L \propto M^{3.5}$$

This relation comes from application of the laws of nuclear physics to stars. Main-sequence stars in the upper left-hand corner of the HR diagram have a very high luminosity and therefore are very massive.

Worked example

D.12 Use the HR diagram and the mass–luminosity relation to estimate the ratio of the density of Altair to that of the Sun.

The two stars have the same radius and hence the same volume. The luminosity of Altair is about 10 times that of the Sun. From

$$\frac{L}{L_\odot} = \left(\frac{M}{M_\odot}\right)^{3.5}$$

we get

$$10 = \left(\frac{M}{M_\odot}\right)^{3.5} \quad \Rightarrow \quad \frac{M}{M_\odot} = 10^{1/3.5} \approx 1.9$$

Hence the ratio of densities is also about 1.9, since the volumes are the same.

D2.7 Cepheid stars

Cepheid variable stars are stars whose luminosity is not constant in time but varies **periodically** from a minimum to a maximum, the periods being typically from a couple of days to a couple of months. The brightness of the star increases sharply and then fades off more gradually, as shown in the **light curve** of a Cepheid in Figure **D.15**.

The mechanism for the periodic variation of the luminosity of Cepheid stars is the following. As radiation rushes outwards it ionises helium atoms in the atmosphere of the star. The freed electrons, through collisions, heat up the star's atmosphere. This increases the pressure, which forces the outer layers of the star to expand. When most of the helium is ionised, radiation now manages to leave the star, and the star cools down and begins to contract under its own weight. This makes helium nuclei recombine with electrons, and so the cycle repeats as helium can again be ionised. The star is brightest when the surface is expanding outwards at maximum speed.

Cepheids occupy a strip between the main sequence and the red giants on an HR diagram.

At the beginning of the 20th century, astronomer Henrietta Leavitt discovered a remarkably precise relationship between the average luminosity of Cepheids and their period. The longer the period, the larger the luminosity (see Figure D.16). This makes Cepheid stars **standard candles** – that is, stars of a known luminosity, obtained by measuring their period.

Exam tip
The reason for a Cepheid star's periodic variation in luminosity is the periodic expansion and contraction of the outer layers of the star.

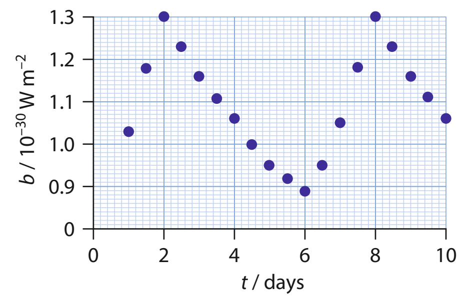


Figure D.15 The apparent brightness of a Cepheid star varies periodically with time.

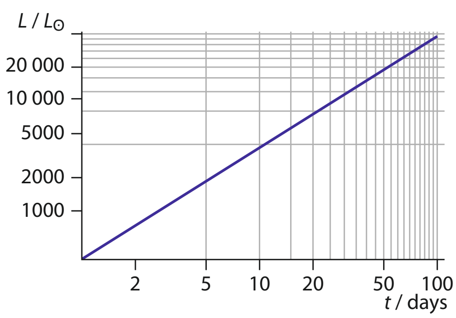


Figure D.16 The relationship between peak luminosity and period for Cepheid stars.

Worked example

D.13 Estimate the distance of the Cepheid whose light curve is shown in Figure **D.15**.

The period is 6 days. From Figure **D.16**, this corresponds to a luminosity of about 2000 solar luminosities, or about $L = 7.2 \times 10^{29}$ W. The average apparent brightness is $b = 1.1 \times 10^{-10}\,\mathrm{W\,m^{-2}}$. Therefore

$$b = \frac{L}{4\pi d^2}$$

$$\Rightarrow d = \sqrt{\frac{L}{4\pi b}} = \sqrt{\frac{7.2 \times 10^{29}}{4\pi \times 1.1 \times 10^{-10}}}\ \mathrm{m}$$

$$= 2.28 \times 10^{19}\,\mathrm{m} \approx 2400\,\mathrm{ly} \approx 740\,\mathrm{pc}$$

Thus, one can determine the distance to the galaxy in which a Cepheid is assumed to be. The Cepheid method can be used to find distances up to a few megaparsecs.

D2.8 Stellar evolution: the Chandrasekhar and Oppenheimer–Volkoff limits

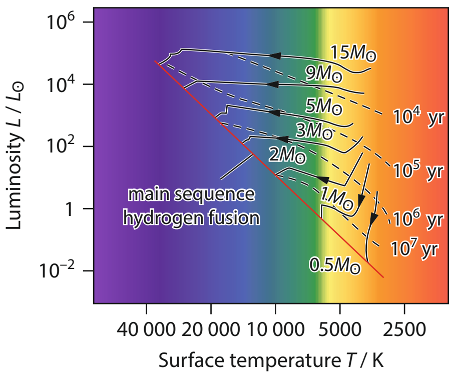


Figure D.17 Evolutionary tracks of protostars as they approach the main sequence. $M_\odot$ stands for one solar mass.

Stars are formed out of contracting gases and dust in the **interstellar medium**, which has hydrogen as its main constituent. Initially the star has a low surface temperature and so its position is somewhere to the right of the main sequence on the HR diagram. As the star contracts under its own weight, gravitational potential energy is converted into thermal energy and the star heats up; it begins to move towards the main sequence. The time taken to reach the main sequence depends on the mass of the star; heavier stars take less time. Our Sun, an average star, has taken about 20 to 30 million years (see Figure **D.17**).

As a star is compressed more and more (under the action of gravity), its temperature rises and so does its pressure. Eventually, the temperature in the core reaches 5×10^6 to 10^7 K and nuclear fusion reactions commence, resulting in the release of enormous amounts of energy. The energy released can account for the sustained luminosity of stars such as our Sun, for example, over the 4–5 billion years of its life so far. Thus, nuclear fusion provides the energy that is needed to keep the star hot, so that its pressure is high enough to oppose further contraction, and at the same time to provide the energy that the star is radiating into space.

On the main sequence, the main nuclear fusion reactions are those of the proton–proton cycle (Section **D1.2**), in which the net effect is to turn four hydrogen nuclei into one helium-4 nucleus.

The mass of a star is the main factor that determines its evolution off the main sequence.

When about 12% of the hydrogen in the star has been used up in nuclear fusion, a series of instabilities develops in the star, upsetting the delicate balance between radiation pressure and gravitational pressure. The star will then begin to move away from the main sequence. What happens next is determined mainly by the mass of the star. Other types of nuclear fusion reactions will take place (see Section **D4**) and the star will change in size and surface temperature (and hence colour).

The changes that take place can be shown as paths on the HR diagram. We may distinguish two essentially different paths, the first for what we will call **low-mass** stars, with a mass less than about eight solar masses, and the second for stars with a higher mass. (In reality the situation is more complex, but this simple distinction is sufficient for the purposes of this course.)

In low-mass stars, helium collects in the core of the star, surrounded by a thin shell of hydrogen and a bigger hydrogen envelope (Figure **D.18**).

Only hydrogen in the thin inner shell undergoes nuclear fusion to helium. The temperature and pressure of the helium build up and eventually helium itself begins to fuse (this is called the 'helium flash'), with helium in a thin inner shell producing carbon in the core. In the core, some carbon nuclei fuse with helium to form oxygen. Oxygen is the heaviest element that can be produced in low-mass stars; the temperature never rises enough for production of heavier elements. The hydrogen in the thin shell is still fusing, so the star now has nuclear fusion in two shells, the H and He shells. The huge release of energy blows away the outer layers of the star in an explosion called a **planetary nebula**; mass is thrown into space, leaving behind the carbon core (and some oxygen). This evolution may be shown on an HR diagram (Figure **D.19**). We will return to the processes in the core in Section **D4.2**.

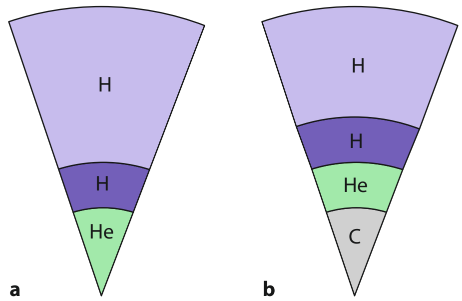


Figure D.18 **a** The structure of a low-mass star after it leaves the main sequence. **b** After helium begins to fuse, carbon collects in the core.

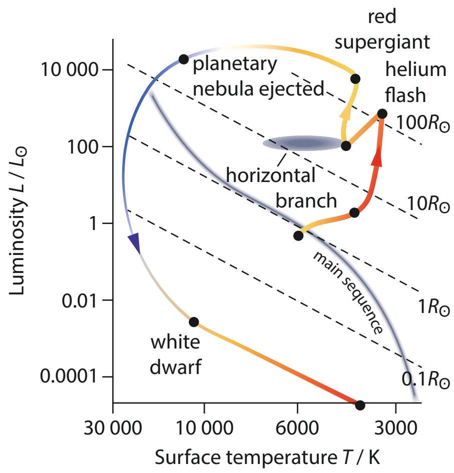


Figure D.19 Evolutionary path of a low-mass star. This is the path of a star of one solar mass that ends up as a white dwarf, which continues to cool down, moving the star ever more to the right on the HR diagram.

The path takes the star off the main sequence and into the red giant region. The star gets bigger and cooler on the surface and hence becomes red in colour. The time taken to leave the main sequence and reach the planetary nebula stage is short compared with the time spent on the main sequence: it takes from a few tens to a few hundreds of million of years. The path then takes the star to the white dwarf region. The star is now a stable but dead star (Figure **D.20**). No nuclear reactions take place in the core.

The conditions in the core mean that the electrons behave as a gas, and the pressure they generate is what keeps the core from collapsing further under its weight. This pressure is called **electron degeneracy pressure** and is the result of a quantum mechanical effect, referred to as the Pauli Exclusion Principle, which states that no two electrons may occupy the same quantum state.

The core has now become a **white dwarf** star. Now exposed, and with no further energy source, the star is doomed to cool down to practically zero temperature and will then become a **black dwarf**.

Figure D.20 The Helix, a planetary nebula. The star that produced this nebula can be seen at its exact centre.

Electron degeneracy pressure prevents the further collapse of the core and – provided the mass of the core is less than about 1.4 solar masses – the star will become a stable white dwarf. This important number is known in astrophysics as the **Chandrasekhar limit**.

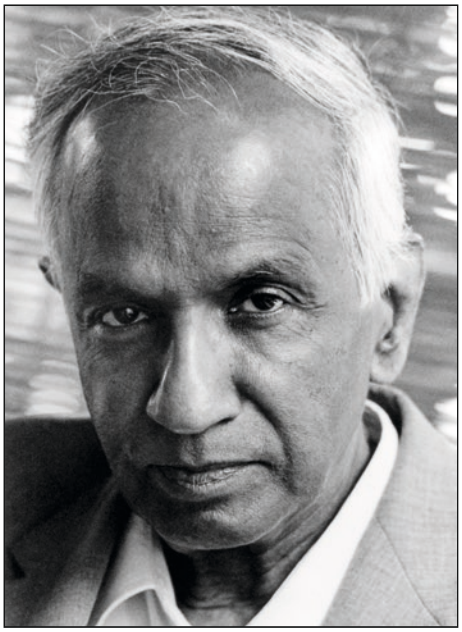


Figure D.21 Subrahmanyan Chandrasekhar (1910–1995).

Exam tip
Fusion ends with the production of iron.

Exam tip
This summary and the paths on the HR diagram are the bare minimum you should know for an exam.

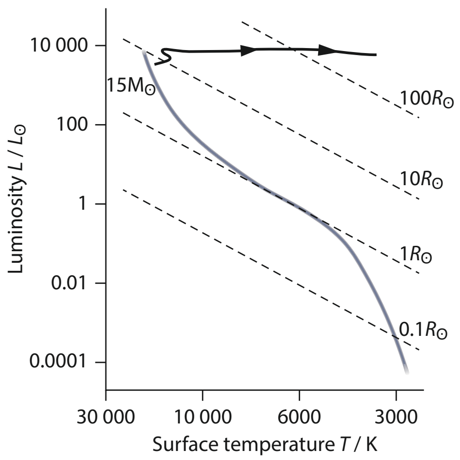


Figure D.22 Evolutionary path of a star of 15 solar masses. It becomes a red supergiant that explodes in a supernova. After the supernova, the star becomes a neutron star, whose luminosity is too small to be plotted on the HR diagram.

The limit is named after the astrophysicist Subrahmanyan Chandrasekhar (Figure **D.21**), who discovered it in the 1930s.

We now look at the evolution of stars whose mass is greater than about eight solar masses. The process begins much the same way as it did for low-mass stars, but differences begin to show when carbon fuses with helium in the core to form oxygen. If the mass of the star is large enough, the pressure caused by gravity is enough to raise the temperature sufficiently to allow the formation of ever-heavier elements: neon, more oxygen, magnesium and then silicon; eventually iron is produced in the most massive stars, and that is where the process stops, since iron is near the peak of the binding-energy curve. It would require additional energy to be supplied for iron to fuse.

The star moves off the main sequence and into the red supergiant area (Figure **D.22**). As the path moves to the right, ever-heavier elements are produced. The star is very hot in the core. Photons have enough energy at these temperatures to split nuclei apart; in about one second (!) millions of years, worth of nuclear fusion is undone. Nuclei are in turn ripped apart into individual protons and neutrons, so that in a very short time the star is composed mainly of protons, electrons, neutrons and photons.

Because of the high densities involved, the electrons are forced into the protons, turning them into neutrons and producing neutrinos that escape from the star ($e^- + p \rightarrow n + \nu_e$). The star's core is now made up almost entirely of neutrons, and is still contracting rapidly. The Pauli Exclusion Principle may now be applied to the neutrons: if they get too close to one another, a pressure develops to prevent them from getting any closer. But they have already done so, and so the entire core now rebounds to a larger equilibrium size. This rebound is catastrophic for the star, creating an enormous shock wave travelling outwards that tears the outer layers of the star apart. The resulting explosion, called a **supernova**, is much more violent than a planetary nebula. The energy loss from this explosion leads to a drastic drop in the temperature of the star, and it begins to collapse.

The core that is left behind, which is more massive than the Chandrasekhar limit, will most likely become a **neutron star**. Neutron pressure keeps such a star stable, provided the mass of the core is not more than about 2–3 solar masses – the **Oppenheimer–Volkoff limit**. If its mass higher than this, it may collapse further and become a black hole.

Table **D.4** shows the temperatures at which various elements participate in fusion reactions.

Element	$T / 10^6$ K	Where
Hydrogen	1–20	Main sequence
Helium	100	Red giant
Carbon	500–800	Supergiant
Oxygen	1000	Supergiant

Table D.4 Temperatures at which various elements participate in fusion reactions.

To summarise:

If the mass of the core of a star is less than the Chandrasekhar limit of about 1.4 solar masses, it will become a stable white dwarf, in which electron pressure keeps the star from collapsing further.

If the core is more massive than the Chandrasekhar limit but less than the Oppenheimer–Volkoff limit of about 2–3 solar masses, the core will collapse further until electrons are driven into protons, forming neutrons. Neutron pressure now keeps the star from collapsing further, and the star becomes a neutron star.

If the Oppenheimer–Volkoff limit is exceeded, the star will become a black hole.

Initial mass of star (in terms of solar masses)	Outcome
0.08–0.25	White dwarf with helium core
0.25–8	White dwarf with carbon core
8–12	White dwarf with oxygen/neon/magnesium core
12–40	Neutron star
>40	Black hole

Table D.5 The final fate of stars with various initial masses.

Table **D.5** shows the final end products of evolution for different initial stellar masses. Figure **D.23** is a schematic summary of the life history of a star.

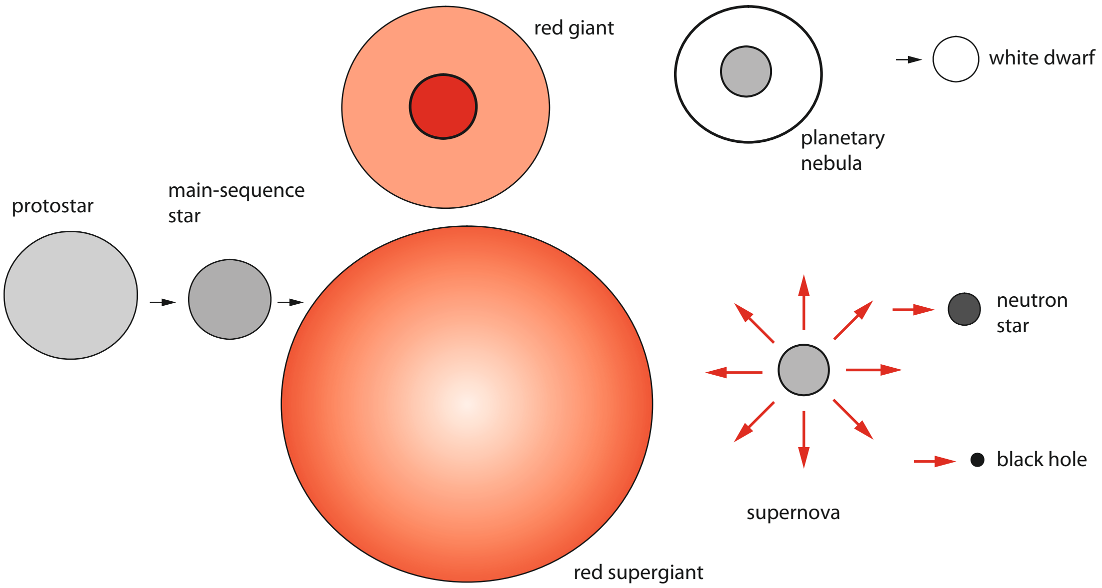


Figure D.23 The birth and death of a star. The star begins as a protostar, evolves to the main sequence and then becomes a red giant or supergiant. After a planetary nebula or supernova explosion, the core of the star develops into one of the three final stages of stellar evolution: white dwarf, neutron star or black hole.

Nature of science

Evidence from starlight

The light from a star is the best source of information about it. The distribution of frequencies tells us its surface temperature, and the actual frequencies present tell us its composition, as each element has a characteristic spectrum. The luminosity and temperature of a star are related, and together give us information about the evolution of stars of different masses. Using this evidence, Chandrasekhar predicted a limit to the mass of a star that would become a white dwarf, while Oppenheimer and Volkoff predicted the mass above which it would become a black hole. The development of theories of stellar evolution illustrates how, starting from simple observations of the natural world, science can build up a detailed picture of how the universe works. Further observations are then needed to confirm or reject hypotheses.

Test yourself

18 Describe how a stellar absorption spectrum is formed.

19 Describe how the chemical composition of a star may be determined.

20 Describe how the colour of the light from a star can be used to determine its surface temperature.

21 Stars A and B emit most of their light at wavelengths of 650 nm and 480 nm, respectively. Star A has twice the radius of star B. Find the ratio of the luminosities of the stars.

22 **a** State what is meant by a Hertzsprung–Russell (HR) diagram.

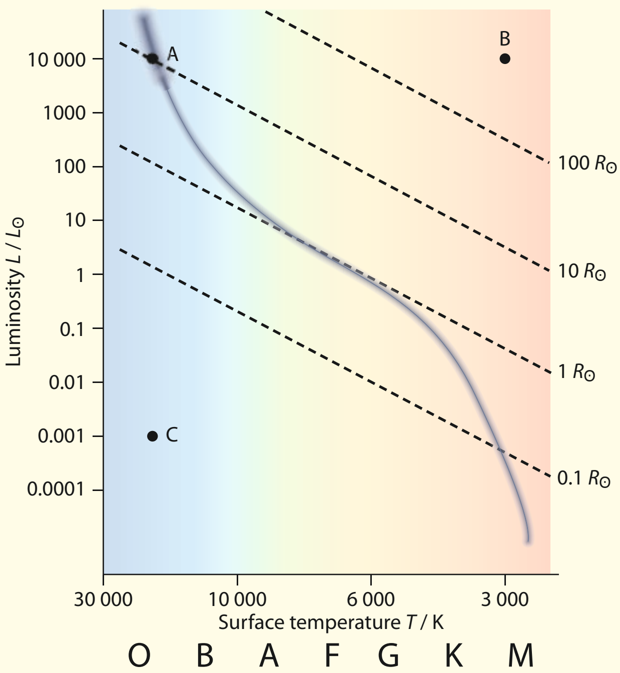
b Describe the main features of the HR diagram.

c The luminosity of the Sun is 3.9×10^{26} W and its radius is 7.0×108 m. For star A in the HR diagram below **calculate**

i the temperature

ii the density in terms of the Sun's density.

d For stars B and C calculate the radius in terms of the Sun's radius.



23 A main-sequence star emits most of its energy at a wavelength of 2.42×10^{-7} m. Its apparent brightness is measured to be 8.56×10^{-12} W m^{-2}. Estimate its distance using the HR diagram in question **22**.

24 A main-sequence star is 15 times more massive than our Sun. Calculate the luminosity of this star in terms of the solar luminosity.

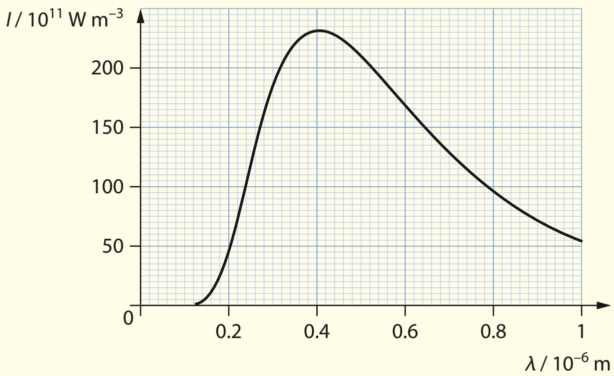
25 **a** The luminosity of a main-sequence star is 4500 times greater than the luminosity of our Sun. Estimate the mass of this star in terms of the solar mass.

b A star has a mass of 12 solar masses and a luminosity of 3200 solar luminosities. Determine whether this could be a main-sequence star.

26 Describe the mechanism by which the luminosity of Cepheid stars varies.

27 Using Figure **D.16**, calculate the distance of a Cepheid variable star whose period is 10 days and whose average apparent brightness is 3.45×10^{-14} W m^{-2}.

28 **a** Find the temperature of a star whose spectrum is shown below.



b Assuming this is a main-sequence star, *estimate* its luminosity using the HR diagram in question **22**.

29 Estimate the temperature of the universe when the peak wavelength of the radiation in the universe was 7.0×10^{-7} m.

30 A neutron star has a radius of 30 km and makes 500 revolutions per second.

a Calculate the speed of a point on its equator.

b Determine what fraction of the speed of light this is.

31 Describe the formation of a red giant star.

32 **a** Describe what is meant by a **planetary nebula**.

b Suggest why most photographs show planetary nebulae as rings: doesn't the gas surround the core in all directions?

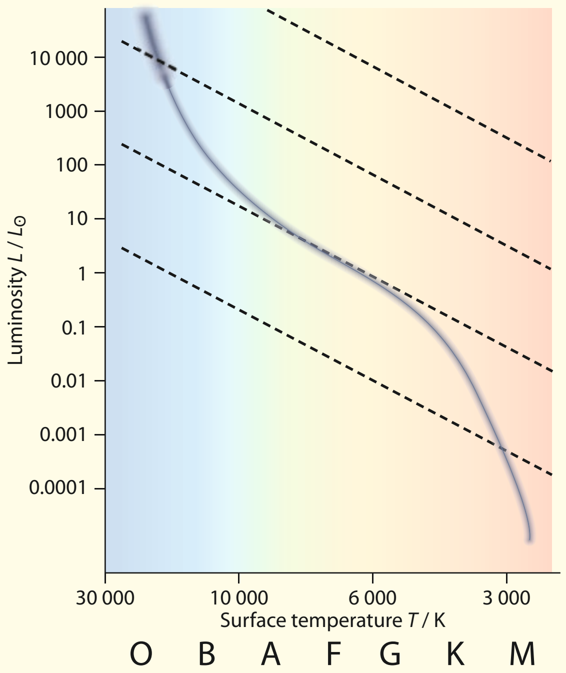
33 Assume that no stars of mass greater than about two solar masses could form anywhere. Would life as we know it on Earth be possible?

34 Describe the evolution of a main-sequence star of mass:

a 2 solar masses

b 20 solar masses.

c Show the evolutionary paths of these stars on a copy of the HR diagram below.



35 **a** Describe the formation of a white dwarf star.

b List two properties of a white dwarf.

c Describe the mechanism which prevents a white dwarf from collapsing under the action of gravity.

36 Describe **two** differences between a main-sequence star and a white dwarf.

37 A white dwarf, of mass half that of the Sun and radius equal to one Earth radius, is formed. Estimate its density.

38 Describe **two** differences between a main-sequence star and a neutron star.

39 **a** Describe the formation of a neutron star.

b List two properties of a neutron star.

c Describe the mechanism which prevents a neutron star from collapsing under the action of gravity.

40 Describe your understanding of the **Chandrasekhar limit**.

41 Describe your understanding of the **Oppenheimer–Volkoff limit**.

42 Assume that the material of a main-sequence star obeys the ideal gas law, $PV = NkT$. The volume of the star is proportional to the cube of its radius R, and N is proportional to the mass M of the star.

a Show that $PR^3 \propto MT$.

The star is in equilibrium under the action of its own gravity, which tends to collapse it, and the pressure created by the outflow of energy from its interior, which tends to expand it. It can be shown that this equilibrium results in the condition $P \propto \frac{M^2}{R^2}$. **(Can you see how?)**

b Combine these two proportionalities to show that $P \propto \frac{M}{R}$. Use this result to explain that, as a star shrinks, its temperature goes up.

c Conclude this rough analysis by showing that the luminosity of main-sequence stars of the same density is given by $L \propto M^{3.3}$.