Investigation 4-1 Circular Orbits

Purpose: To investigate factors involved in the uniform circular motion of a mass revolving at the end of a string.

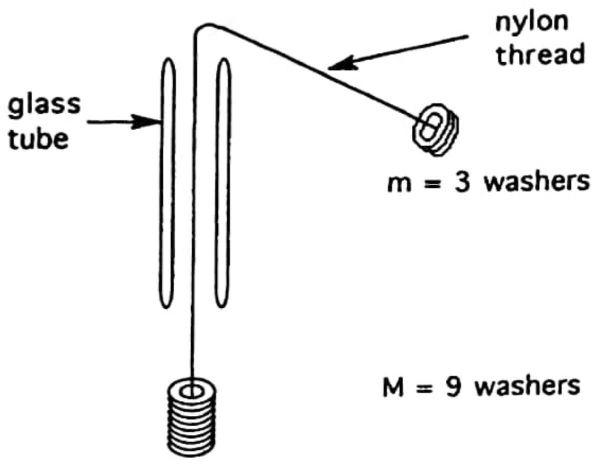

Figure 4.9

Introduction

Some aspects of circular motion can be studied using the simple equipment shown in **Figure 4.9**. A small mass m (a bundle of 3 washers) 'orbits' on the end of a string. A large mass M experiences a force of gravity Mg. In this ***Investigation***, M is chosen to be a bundle of 9 identical washers, so that $M = 3m$.

The glass tube is smoothly polished at the top, and strong, smooth nylon thread joins the two masses. In effect, the edge of the tube acts like a pulley, changing the direction of the tension force in the string without changing its magnitude.

When the small mass m is made to 'orbit' around the top end of the glass tube, mass M remains static, so the tension in the string along its length L is equal to the downward force of gravity on M, which is Mg.

The radius of the orbit is not L, however, but R (the horizontal distance from m to the vertical glass tube). See **Figure 4.10**.

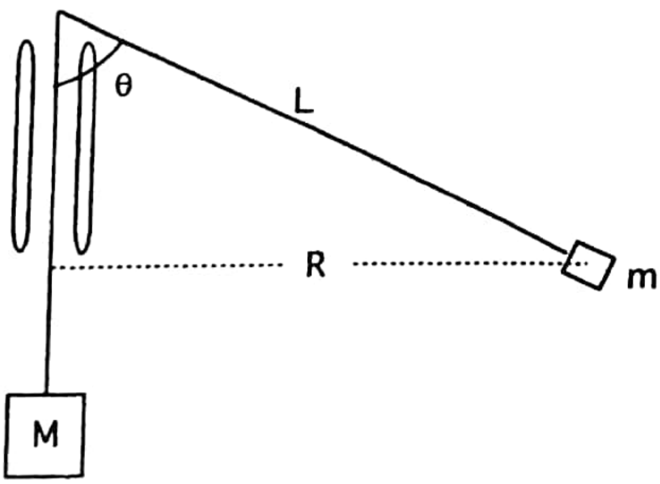

Figure 4.10

When the small mass m is made to 'orbit' the top of the glass tube, we will assume the upward force on the stationary mass M has a magnitude of Mg. The reaction force exerted on m through the string has the same magnitude (Mg) but is exerted in the opposite direction. In **Figure 4.11**, this tension force is labelled $T = Mg$. Consider the two *components* of $\boldsymbol{T}$. The vertical component $\boldsymbol{F_y}$, balances the downward force of gravity on m, so $\boldsymbol{F_y} = mg$. The horizontal component of $\boldsymbol{T}$ is the **centripetal force,** so $\boldsymbol{F_x} = \boldsymbol{F_c}$.

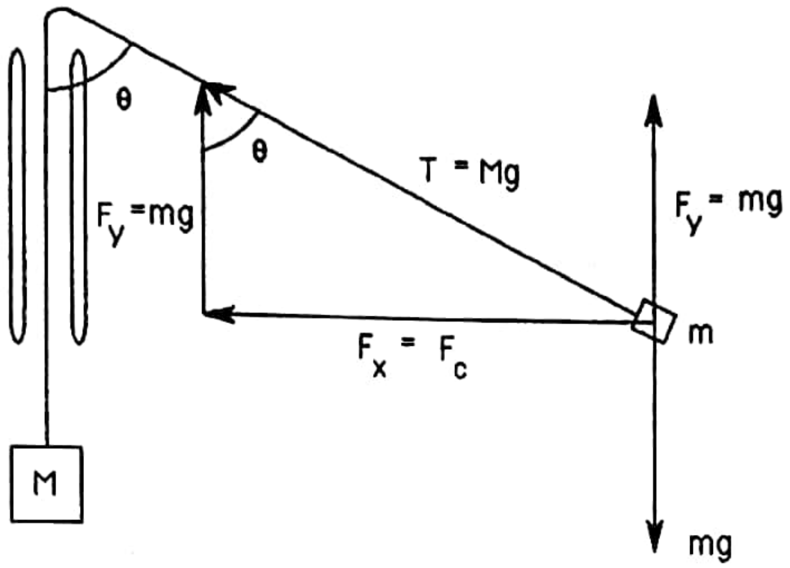

Figure 4.11

Procedure

1. Prepare the apparatus, using 3 identical washers for the orbiting mass and 9 washers for the large mass. ***Predict*** what the angle θ between the thread and the vertical glass tube will be, using the fact that

$$\text{cosine } \theta = \frac{F_y}{T} = \frac{mg}{Mg} = \frac{m}{M}.$$

2. In this investigation, you will vary *L* and measure the period of revolution *T* for each length *L*. Measure from the centre of gravity of the mass *m* along the string, and mark off distances of 20 cm, 40 cm, 60 cm, 80 cm and 100 cm with chalk or a small dab of correction fluid.

3. Hold the glass tube vertically in your hand, and swing mass *m* until it achieves a stable orbit such that length *L* is 20.0 cm. While you keep the orbiting mass revolving, have your partner make the following measurements (as best he can, for it will be difficult!):

   (a) the radius of the orbit, *R*. Your partner will have to hold a metre stick as near to the orbiting mass as he dares (to avoid decapitation) and estimate *R* as closely as possible;

   (b) the period of revolution, *T*. Your partner will time how long the mass takes to make ten revolutions, then divide the total time by 10.

4. Repeat **Procedure 3** for lengths of 40.0 cm, 60.0 cm, 80.0 cm and 100.0 cm. Record all your measurements in a table like **Table 1**.

5. Calculate the ratio of *R/L* . From **Figure 4.10** you can see that this ratio equals sine θ. Calculate θ knowing sine θ.

Table 1 Data for *Investigation* 6

Length of String, L [cm]	Radius of Orbit, R [cm]	Period of Revolution, T, [s]	$\sin\theta = \frac{R}{L}$	θ [°]	$\cos\theta = \frac{m}{M}$	θ [°]
20.0						
40.0						
60.0						
80.0						
100.0						

6. Plot a graph with T on the Y-axis (since it is the *dependent variable)* and L on the X-axis (since it is the *independent variable*).
7. Examine the shape of your graph of T vs L, and make an educated guess at what the power law might be. For example, if you think the power law is most likely $T^n = k \cdot L$, then plot a graph of T^n vs L. If you choose the correct value of n, your graph should be a straight line passing through (0,0).
8. Determine the slope of your final, straight-line graph, and express it in appropriate units. Then write an **equation** for your straight line, incorporating the **slope.**
9. You can use the results of your experiment to do a check on the formula for centripetal force.

Since $F_C = \dfrac{m4\pi^2 R}{T^2}$, therefore, $T^2 = \dfrac{m4\pi^2 R}{F_C}$.

Figure 4.10 shows you that $R = L\sin\theta$, and **Figure 4.11** makes it clear that $F_C = F_X = Mg\sin\theta$. Therefore, we can write

$$T^2 = \frac{m4\pi^2 L\sin\theta}{Mg\sin\theta} = \frac{m4\pi^2}{Mg}L.$$

Thus, the **theoretical** slope of a graph of T^2 vs L should be $\dfrac{m4\pi^2}{Mg}$.

Concluding Questions

1. Compare the average value of angle θ calculated from $\sin\theta = R/L$ with the value of θ predicted using cosine $\theta = m/M$. What is the **percent difference** between the two results?
2. According to your results, how does the period, T, of the orbiting mass vary with the length, L? Write a **specific equation** for your graphical result.
3. What is the **theoretical value** of your slope for the graph in **Concluding Question 2**? What is the **percent difference** between your graph's actual slope and the theoretical slope?
4. Discuss **sources of error** in this experiment and how you might be able to reduce their effects.