D5 Further cosmology (HL)

This section deals with some open questions in cosmology, questions that are the subject of intensive current research. These include the evidence for and the nature of dark matter and dark energy, fluctuations in the CMB, and the **rotation curves** of galaxies

Learning objectives

- Describe the cosmological principle.
- Understand the fluctuations in the CMB.
- Understand the cosmological origin of red-shift.
- Derive the critical density and understand its significance.
- Describe dark matter.
- Derive rotation curves and understand how they provide evidence for dark matter.
- Understand dark energy.
- Sketch the variation of the scale factor with time for various models.

D5.1 The cosmological principle

The universe appears to be full of structure. There are planets and moons in our solar system, there are stars in our galaxy, our galaxy is part of a **cluster of galaxies** and our cluster is part of an even bigger **supercluster** of galaxies.

If we look at the universe on a very large scale, however, we no longer see any structure. If we imagine cutting up the universe into cubes some 300 Mpc on a side, the interior of any one of these cubes would look much the same as the interior of any other, anywhere else in the universe. This is an expression of the so-called **homogeneity principle** in cosmology: on a large enough scale, the universe looks uniform.

Similarly, if we look in different directions, we see essentially the same thing. If we look far enough in any direction, we will count the same number of galaxies. No one direction is special in comparison with another. This leads to a second principle of cosmology, the **isotropy principle**. A related observation is the high degree of isotropy of the CMB.

These two principles, homogeneity and isotropy, make up what is called the **cosmological principle**, which has had a profound role in the development of models of cosmology.

The cosmological principle implies that the universe has no edge (for if it did, the part of the universe near the edge would look different from a part far from the edge, violating the homogeneity principle). Similarly, it implies that the universe has no centre (for if it did, an observation from the centre would show a different picture from an observation from any other point, violating the principle of isotropy).

D5.2 Fluctuations in the CMB

We have noted several times that the CMB is uniform and isotropic. However, it is not perfectly so. There are small variations ΔT in temperature, of the order of $\frac{\Delta T}{T} \approx 10^{-5}$, where $T = 2.723\,\mathrm{K}$ is the average temperature. These variations in temperature are related to variations in the density of the universe. In turn, variations in density are the key to how structures formed in the universe. With perfectly uniform temperature and density in the universe, stars and galaxies would not form.

The anisotropies in the CMB are crucial in understanding the formation of structures.

In addition to helping us understand structures, CMB anisotropies are related to the geometry of the universe. There would be different degrees of anisotropy depending on whether the universe has positive, zero or negative curvature (see the section on the dependence of the **scale factor** on time later in this section). A number of investigations of CMB anisotropies have been carried out, using COBE, WMAP, the Planck satellite observatory and the Boomerang (Balloon Observations

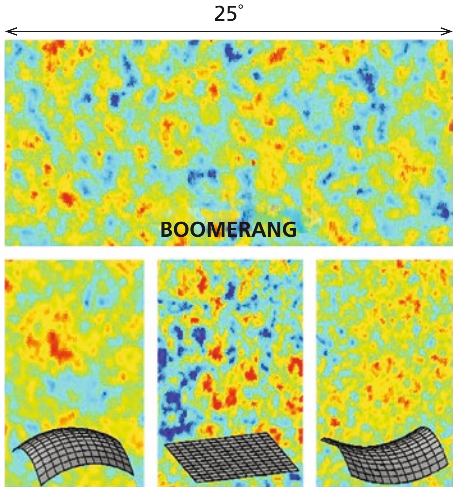


Figure D.33 Fluctuations in temperature are shown as differences in colour in this image from the Boomerang collaboration. Theoretical models using space of different curvatures are also shown. There is a clear match with the flat-universe case. © The Boomerang Collaboration.

Of Millimetric Extragalactic Radiation) collaboration. Figure **D.33** shows fluctuations in the CMB temperature obtained by the Boomerang collaboration, and three theoretical predictions of what that anisotropy should look like in models with positive, negative and zero **curvature** of space. Different colours correspond to different temperatures. Even judged by eye, the data appear to be consistent with the flat case.

Figure **D.34** is a spectacular map from the Planck satellite observatory, showing **CMB fluctuations** in temperature as small as a few millionths of a degree. This is a map of the radiation filling the universe when it was only about 380 000 years old.

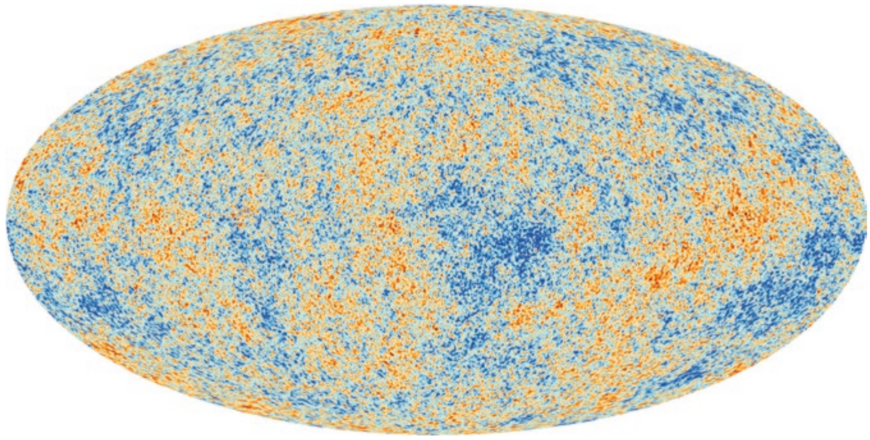


Figure D.34 Fluctuations in temperature of the CMB according to ESA's Planck satellite observatory. (©ESA and the Planck Collaboration, reproduced with permission)

Studies of CMB anisotropy also give crucial information on cosmological parameters such as the density of matter and energy in the universe.

D5.3 Rotation curves and the mass of galaxies

Consider a planet as it revolves about the Sun (Figure **D.35a**). In Topics **6** and **10** we determined, using $\frac{GMm}{r^2}=\frac{mv^2}{r}$, that the speed of a planet a distance r from the Sun is $v=\sqrt{\frac{GM}{r}}$. This means that $v\propto\frac{1}{\sqrt{r}}$. Plotting rotational speed against distance gives what is called a **rotation curve**, as shown in Figure **D.35b**.

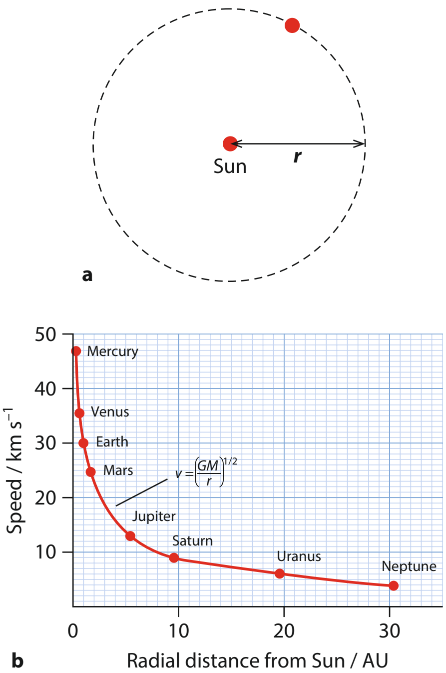


Figure D.35 **a** A particle orbiting a central mass. **b** The rotation curve of the particle in **a** shows a characteristic drop. (From M. Jones and R. Lambourne, An Introduction to Galaxies and Cosmology, Cambridge University Press, in association with The Open University, 2004. © The Open University, used with permission)

Now consider a spherical mass cloud of uniform density (Figure **D.36a**). What is the speed of a particle rotating about the centre at a distance r? We can still use $v=\sqrt{\frac{GM}{r}}$, but now M stands for the mass in the spherical body up to a distance r from the centre. Since the density is constant we have that

$$\rho=\frac{M}{V}=\frac{M}{\frac{4\pi r^3}{3}}=\frac{3M}{4\pi r^3}$$

and hence

$$M=\frac{4\pi r^3\rho}{3}$$

Thus

$$v = \sqrt{\frac{G4\pi r^3 \rho}{3r}}$$

and so $v \propto r$. The rotation curve is a straight line through the origin. This is valid for r up to R, the radius of the spherical mass cloud. Beyond R the curve is like that of Figure **D.35**.

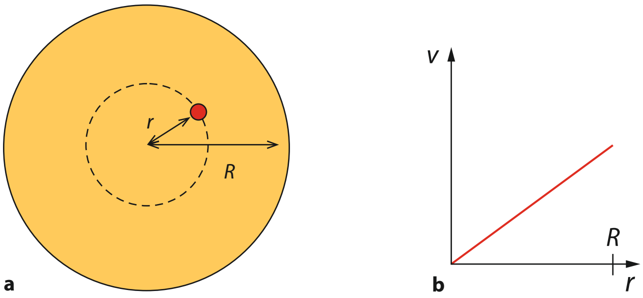


Figure D.36 **a** A particle orbiting around the centre of a uniform spherical cloud. **b** The rotation curve of the particle in **a**.

Worked example

D.23 Consider a system in which the mass varies with distance from the axis according to $M = kr$, where k is a constant. Derive the rotation curve for such a system.

We start with $v = \sqrt{\frac{GM}{r}}$. We get $v = \sqrt{\frac{Gkr}{r}} = \sqrt{Gk} =$ constant. The rotation curve would thus be a horizontal straight line.

Figure **D.37** shows the rotation curve of the Milky Way galaxy.

This rotation curve is not one that belongs to a large central mass, as in Figure **D.35**. Its main feature is the flatness of the curve at large distances from the centre. Notice that the flatness starts at distances of about 13 kpc. The central galactic disc has a radius of about 15 kpc. This means that there is substantial mass outside the central galactic disc. Furthermore, according to Worked example **D.23**, a flat curve corresponds to increasing mass with distance from the centre.

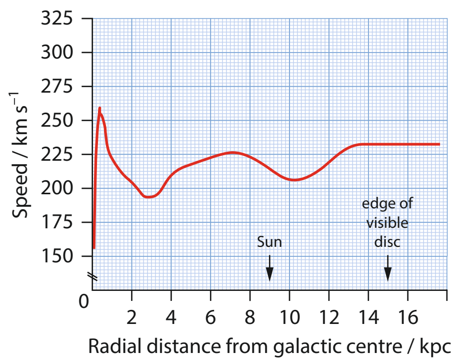


Figure D.37 The rotation curve of our galaxy shows a flat region, indicating the presence of matter far from the galactic disc. (Adapted from Combes, F. (1991) Distribution of CO in the Milky Way, Annual Review of Astronomy and Astrophysics, 29, pp.195–237)

> The flat part of the galaxy's rotation curve indicates substantial mass far from the centre.

If the mass were contained within a given distance, then past that distance the rotation curve should have dropped, as it does in Figure **D.35**. The problem is that no such drop is seen – but at the same time no such mass is visible. Arguments like this have led to the conclusion that there must be considerable mass at large distances past the galactic disc. This is **dark matter**: matter that is too cold to radiate and so cannot be seen. It is estimated that, in our galaxy, dark matter forms a spherical halo around the galaxy and has a mass that is about 10 times larger than the mass of all the stars in the galaxy!

D5.4 Dark matter

It is estimated that 85% of the matter in the universe is dark matter. It cannot be seen; we know of its existence mainly from its gravitational effects on nearby bodies.

What could dark matter be? It could be ordinary, cold matter that does not radiate – like, for example, brown dwarfs, black dwarfs or small planets. Collectively these are called MACHOs (MAssive Compact Halo Objects). This is matter consisting mainly of protons and neutrons, so it also called **baryonic matter**. The problem is that there are limits to how much baryonic matter there can be. The limit is at most 15%, so dark matter must also contain other, more exotic forms.

The class of non-baryonic objects which are candidates for dark matter are called WIMPs (for Weakly Interacting Massive Particles). Neutrinos fall into this class since they are known to have a small mass, although their tiny mass is not enough to account for all non-baryonic dark matter. Unconfirmed theories of elementary particle physics based mainly on the idea of **supersymmetry** (a proposed symmetry between particles with integral spin and particles with half-integral spin) predict the existence of various particles that would be WIMP candidates – but no such particles have been discovered.

So the answer to the question 'what is dark matter?' is mainly unknown at the moment.

D5.5 The cosmological origin of red-shift

In Section **D3.2** we derived the formula

$$\frac{\lambda}{\lambda_0}=\frac{R}{R_0}$$

where R_0 is the value of the scale factor at the time of emission of a photon of wavelength λ_0, R is the value of the scale factor at the present time (when the photon is received) and λ is the wavelength of the photon as measured at the present time. This gives a cosmological interpretation of the red-shift, rather than one based on the Doppler effect: the space in between us and the galaxies is stretching, so wavelengths stretch as well.

A direct consequence of this is on the temperature of the CMB radiation that fills the universe. The wavelength λ_0 corresponds to a CMB temperature of T_0. By the Wien displacement law,

$$\lambda_0 T_0 = \lambda T = \text{constant}$$

Therefore

$$\frac{\lambda}{\lambda_0}=\frac{T_0}{T}$$

This implies that

$$\frac{T_0}{T}=\frac{R}{R_0} \text{ or } T \propto \frac{1}{R}$$

This shows that, as the universe expands (that is, as R gets bigger), the temperature drops. This is why the universe is cooling down, and why the present temperature of the CMB is so low (2.7 K).

Worked example

D.24 The photons of CMB radiation observed today are thought to have been emitted at a time when the temperature of the universe was about 3.0×10^3 K. Estimate the size of the universe then compared with its size now.

From $T \propto \frac{1}{R}$ we find $\frac{T_0}{T} = \frac{R}{R_0}$, so $\frac{R}{R_0} = \frac{3.0 \times 10^3}{2.7} \approx 1100$, so the universe then was about 1100 times smaller.

One of the great problems in cosmology is to determine how the scale factor depends on time. We will look at this problem in the next two sections.

D5.6 Critical density

We begin the discussion in this section by solving a problem in Newtonian gravitation. Consider a spherical cloud of dust of radius r and mass M, and a mass m at the surface of this cloud which is moving away from the centre with a velocity v satisfying Hubble's law, $v = H_0 r$; see Figure **D.38**.

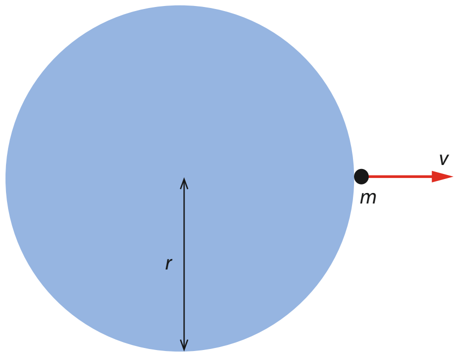


Figure D.38 Estimating critical density.

The total energy of the mass m is

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

If ρ is the density of the cloud, then $M = \rho\frac{4}{3}\pi r^3$. Using this together with $v = H_0 r$, we find

$$E = \frac{1}{2}mr^2\left(H_0^2 - \frac{8\pi\rho G}{3}\right)$$

The mass m will continue to move away if its total energy is positive. If the total energy is zero, the expansion will halt at infinity; if it is negative, contraction will follow the expansion. The sign of the term $\left(H_0^2 - \frac{8\pi\rho G}{3}\right)$, that is, the value of ρ relative to the quantity

$$\rho_c = \frac{3H_0^2}{8\pi G} \approx 10^{-26}\,\text{kg}\,\text{m}^{-3}$$

determines the long-term behaviour of the cloud.

This quantity is known as the **critical density**. We have only derived it in a simple setting based on Newton's gravitation. What does it have to do with the universe? We discuss this in the next section.

Exam tip
You must be able to derive the formula for the critical density.

D5.7 The variation of the scale factor with time

In 1915 Einstein published his general theory of relativity, replacing Newton's theory of gravitation with a revolutionary new theory in which the rules of geometry are dictated by the distribution of mass and energy in the universe. Applying Einstein's theory to the universe as a whole results in equations for the scale factor R, and solving these gives the dependence of R on time. Einstein himself believed in a static universe – that is, a universe with R = constant. His equations, however, did not give a constant R. So he modified them, adding his famous **cosmological constant** term, Λ, to make R constant (see Figure **D.39**).

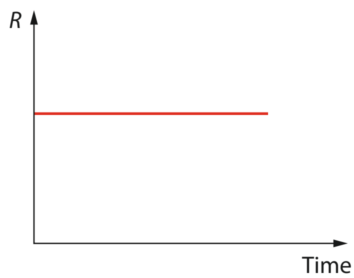


Figure D.39 A model with a constant scale factor. Einstein introduced the cosmological constant Λ in order to make the universe static.

In this model there is no Big Bang, and the universe always has the same size. This was before Hubble discovered the expanding universe. Einstein missed the great chance of theoretically predicting an expanding universe before Hubble; he later called adding the cosmological constant 'the greatest blunder of [his] life'. This constant may be thought of as related to a 'vacuum energy', energy that is present in all space. The idea fell into obscurity for many decades but it did not go away: it was to make a comeback with a vengeance much later! It is now referred to as **dark energy**.

The first serious attempt to determine how R depends on time was made by the Russian mathematician Alexander Friedmann (1888–1925). Friedmann applied the Einstein equations and realised that there were a number of possibilities: the solutions depend on how much matter and energy the universe contains.

We define the **density parameters** Ω_m and Ω_Λ, for matter and dark energy respectively, as

$$\Omega_m = \frac{\rho_m}{\rho_c} \text{ and } \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c}$$

where ρ_m is the actual density of matter in the universe, ρ_c is the critical density derived in Section **D5.6** and ρ_Λ is the density of dark energy. The Friedmann equations give various solutions depending on the values of Ω_m and Ω_Λ. Deciding which solution to pick depends crucially on these values, which is why cosmologists have expended enormous amounts of energy and time trying to accurately measure them.

Figure **D.40** is a schematic representation of the possibilities; the subscript 0 indicates the values of these parameters at the present time. There are four regions in the diagram. The shape of the graph of scale factor versus time is different from region to region.

Notice the red dashed line: for models above the line, the geometry of the universe resembles that of the surface of a sphere. Those below the line have a geometry like that of the surface of a saddle. Points on the line imply a flat universe in which the rules of Euclidean geometry apply. Figure **D.41** illustrates these three models.

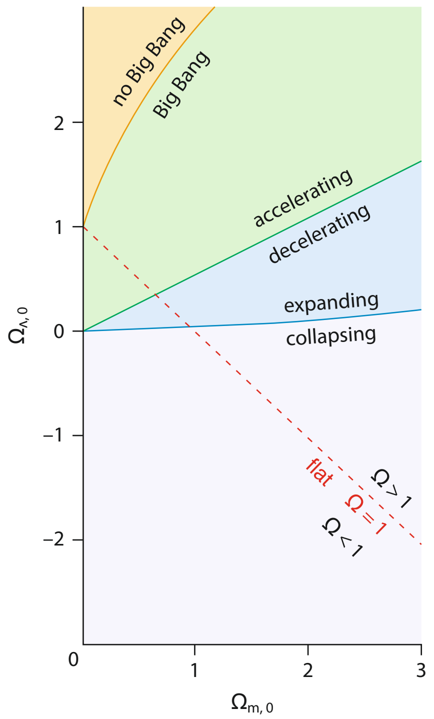


Figure D.40 There are various possibilities for the evolution of the universe depending on how much energy and mass it contains. (From M. Jones and R. Lambourne, An Introduction to Galaxies and Cosmology, Cambridge University Press, in association with The Open University, 2004. © The Open University, used with permission.)

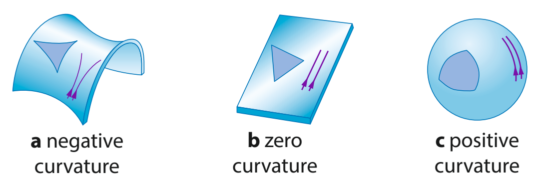


Figure D.41 Three models with different curvatures. **a** In negative-curvature models, the angles of a triangle add up to less than 180° and initially parallel lines eventually diverge. **b** Ordinary flat (Euclidean) geometry. **c** Positive curvature, in which the angles of a triangle add up to more than 180° and initially parallel lines eventually intersect. (From M. Jones and R. Lambourne, An Introduction to Galaxies and Cosmology, Cambridge University Press, in association with The Open University, 2004. © The Open University, used with permission)

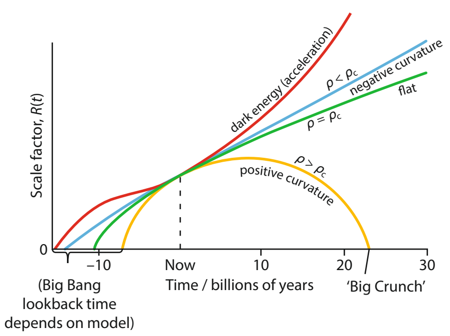


Figure D.42 Solutions of Einstein's equations for the evolution of the scale factor. The present time is indicated by 'now'. Notice that the estimated age of the universe depends on which solution is chosen. Three models assume zero dark energy; the one shown by the red line does not.

Of the very many possibilities, we will be interested in just four cases. The first three correspond to $\Omega_\Lambda = 0$ (they are of mainly historical interest, because observations favour $\Omega_\Lambda \neq 0$). These are shown as the orange, green and blue lines in Figure **D.42**.

In all three cases the scale factor starts from zero, implying a Big Bang. In one possibility (orange line), $R(t)$ starts from zero, increases to a maximum value and then returns to zero; that is, the universe collapses after an initial period of expansion. This is called the **closed model**, and corresponds to $\Omega_m > 1$, i.e. $\rho_m > \rho_c$. The second possibility corresponds to $\Omega_m < 1$, i.e. $\rho_m < \rho_c$. Here the scale factor $R(t)$ increases without limit – the universe continues to expand forever. This is called the **open model**. The third possibility is that the universe expands forever, but with a decreasing rate of expansion, becoming zero at infinite time. This is called the **critical model** and corresponds to $\Omega_m = 1$. The density of the universe in this case is equal to the critical density: $\rho_m = \rho_c$.

Keep in mind that these three models have $\Omega_\Lambda = 0$ and so are *not* consistent with observations. The fourth case, the red line in Figure **D.42**, is the one that agrees with current observations. Data from the Planck satellite observatory (building on previous work by WMAP, Boomerang and COBE) indicate that $\Omega_m \approx 0.32$ and $\Omega_\Lambda \approx 0.68$. This implies that $\Omega_m + \Omega_\Lambda \approx 1$, and corresponds to the red dashed line in Figure **D.42**. This is consistent with the analysis of the Boomerang data that we discussed in Section **D5.2**, and means that at present our universe has a flat geometry and is expanding forever at an accelerating rate, and that 32% of its mass–energy content is matter and 68% is dark energy.

D5.8 Dark energy

In Section **D3.5** we saw how analysis of distant Type Ia supernovae led to the conclusion that the expansion of the universe is accelerating. This ran contrary to expectations: gravity should be slowing the distant galaxies down. We also saw that an accelerating universe demands a non-zero value of the cosmological constant, which in turn implies the presence of a 'vacuum energy' that fills all space; this energy has been called dark energy.

The presence of this energy creates a kind of repulsive force that not only counteracts the effects of gravity on a large scale but actually dominates over it, causing acceleration in distant objects rather than the expected deceleration. The domination of the effects of dark energy over gravity appears to have started about 5 billion years ago.

There is now convincing evidence that $\Omega_m + \Omega_\Lambda \approx 1$, based on detailed studies of anisotropies in CMB radiation undertaken by COBE, WMAP, the Boomerang collaboration and the Planck satellite observatory. Data from Planck indicate that the mass–energy density of the universe consists of approximately 68% dark energy, 27% dark matter and 5% ordinary matter. This means that we understand just 5% of the mass–energy of the universe! These facts – along with the discovery (announced in March 2014 but still not confirmed) of gravitational waves, supporting another important part of the Big Bang model (inflation) – make these very exciting times for cosmology!

Nature of science

Cognitive bias

When interpreting experimental results, it is tempting to dismiss or find ways to explain away results that do not fit with the hypotheses. In the late 20th century, most scientists believed that the expansion of the universe must be slowing down because of the pull of gravity.

Evidence from the analysis of Type Ia supernovae in 1998 showed that the expansion of the universe is accelerating – a very unexpected result. Corroboration from other sources has led to the acceptance of this result, with the proposed dark energy as the cause of the acceleration.

Test yourself

86 **a** Outline what is meant by the **cosmological principle**.

b Explain how this principle may be used to deduce that the universe:

i has no centre

ii has no edge.

87 State **two** pieces of observational evidence that support the cosmological principle.

88 **a** Outline what is meant by the **scale factor of the universe**.

b Sketch a graph to show how the scale factor of the universe varies with time for a model with zero cosmological constant and a density greater than the critical density.

c Draw another graph to show the variation of the CMB temperature for the model in **b**.

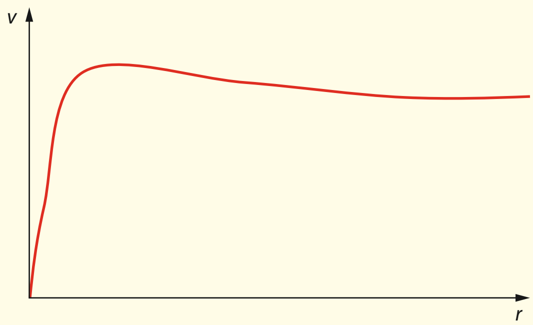
89 Sketch and label **three** graphs to show how the scale factor of the universe varies with time for models with zero cosmological constant. Use your graphs to explain why the three models imply different ages of the universe.

90 **a** Derive the rotation curve formula (showing the variation of speed with distance) for a mass distribution with a uniform density.

b Draw the rotation curve for **a**.

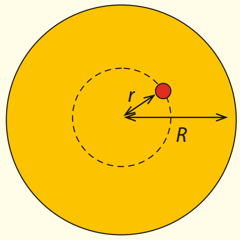
91 **a** Derive the rotation curve formula for a spherical distribution of mass in a galaxy that varies with the distance r from the centre according to $M(r) = kr$, where k is a constant.

b By comparing your answer with the rotation curve below, suggest why your rotation curve formula implies the existence of matter away from the centre of the galaxy.

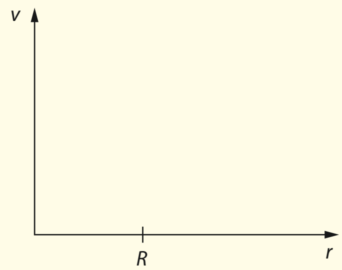


92 Sketch a graph to show the variation of the scale factor for a universe with a non-zero cosmological constant.

93 The diagram below shows a spherical cloud of radius R whose mass is distributed with constant density.



a A particle of mass m is at distance r from the centre of the cloud. On a copy of the axes below, draw a graph to show the expected variation with r of the orbital speed v of the particle for $r < R$ and for $r > R$.



b Describe one way in which the rotation curve of our galaxy differs from your graph.

94 **a** What do you understand by the term **dark matter**?

b Give three possible examples of dark matter.

95 Distinguish between **dark matter** and **dark energy**.

96 Explain why, in an accelerating universe, distant supernovae appear dimmer than expected.

97 **a** Outline what is meant by the **anisotropy of the CMB**.

b State what can be learned from studies of CMB anisotropies.

98 State how studies of CMB anisotropy lead to the conclusion that the universe is flat.

99 Use Figure **D.40** to suggest whether current data support a model with a negative cosmological constant.

100 Derive the dependence $T \propto \frac{1}{R}$ of the temperature T of the CMB on the scale factor R of the universe.

101 **a** Outline what is meant by the critical density.

b Show using Newtonian gravitation that the critical density of a cloud of dust expanding according to Hubble's law is given by $\rho_c = \frac{3H^2}{8\pi G}$.

c Current data suggest that $\Omega_m = 0.32$ and $H_0 = 68\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$. alculate the matter density of the universe.

102 **a** Suggest why determination of the mass density of the universe is very difficult.

b Estimate how many hydrogen atoms per m^3 cubic metre a critical density of $\rho_c \approx 10^{-26}\,\text{kg}\,\text{m}^{-3}$ represents.

103 **a** State what is meant by an accelerating of the universe.

b Draw the variation of the scale factor with time for an accelerating model.

c Use your answer to draw the variation of temperature with time in an accelerating model.

104 Explain, with the use of two-dimensional examples if necessary, the terms **open** and **closed** as they refer to cosmological models. Give an example of a space that is finite without a boundary and another that is finite with a boundary.

105 The density parameter for dark energy is given by $\rho_\Lambda = \frac{\Lambda c^2}{3H_0^2}$. Deduce the value of the cosmological constant, given $\Omega_\Lambda = 0.68$ and $H_0 = 68\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$.

106 **a** List three reasons why Einstein's prediction of a constant scale factor is not correct.

b Identify a point on the diagram in Figure **D.40** where Einstein's model is located.

107 The Friedmann equation states that

$$H^2 = \frac{8\pi G}{3}\left(\rho + \frac{\Lambda c^2}{8\pi G}\right) - \frac{kc^2}{R^2}$$

where the parameter k determines the curvature of the universe: $k > 0$ implies a closed universe, $k < 0$ an open universe and $k = 0$ a flat universe. Deduce the geometry of the universe given that $\Omega_m + \Omega_\Lambda = 1$.

Exam-style questions

1 **a** Describe what is meant by:

i luminosity **[1]**

ii apparent brightness. **[1]**

Achernar is a main-sequence star with a mass equal to 6.7 solar masses. Its apparent brightness is $1.7 \times 10^{-8}\,\text{W}\,\text{m}^{-2}$ and its surface temperature is 2.6 times the Sun's temperature. The luminosity of the Sun is $3.9 \times 10^{26}\,\text{W}$.

b State **one** characteristic of main-sequence stars. **[1]**

c Estimate for Achernar:

i its luminosity **[2]**

ii its parallax angle **[2]**

iii its radius in terms of the solar radius. **[3]**

d **i** Describe the **method of parallax** for measuring distances to stars. **[4]**

ii Suggest whether the parallax method can be used for Achernar. **[1]**

2 **a** Suggest how the chemical composition of a star may be determined. **[3]**

b Explain why stars of the following spectral classes do not show any hydrogen absorption lines in their spectra:

i spectral class O **[2]**

ii spectral class M. **[2]**

c State **one** other property of a star that can be determined from its spectrum. **[1]**

3 **a** Outline the mechanism by which the luminosity of Cepheid stars varies. **[3]**

b Describe how Cepheid stars may be used to estimate the distance to galaxies. **[4]**

c The apparent brightness of a particular Cepheid star varies from $2.4 \times 10^{-8}\,\mathrm{W\,m^{-2}}$ to $3.2 \times 10^{-8}\,\mathrm{W\,m^{-2}}$ with a period of 55 days. Determine the distance to the Cepheid. The luminosity of the Sun is $3.9 \times 10^{26}\,\mathrm{W}$. **[3]**

4 A main-sequence star has a mass equal to 20 solar masses and a radius equal to 1.2 solar radii.

a Estimate:

i the luminosity of this star **[2]**

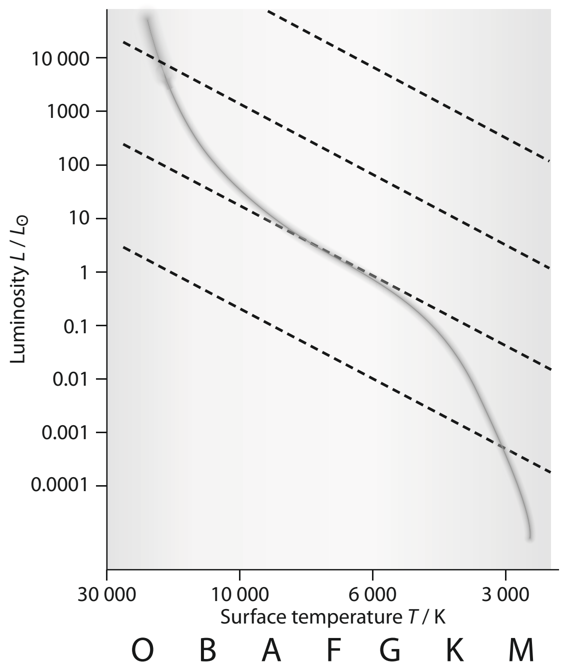
ii the ratio $\frac{T}{T_\odot}$ of the temperature of the star to that of the Sun. **[2]**

b **i** State **two** physical changes the star will undergo after it leaves the main sequence and before it loses any mass. **[2]**

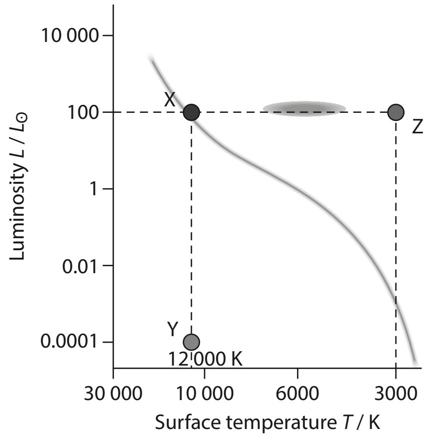
ii The star will eject mass into space in a supernova explosion. Suggest whether this will be a Type Ia or Type II supernova. **[2]**

iii Describe the final equilibrium state of this star. **[3]**

c On a copy of the HR diagram, draw the evolutionary path of the star. **[1]**



5 The HR diagram below shows three stars, X, Y and Z.



a On a copy of the diagram, identify:

i the main sequence **[1]**

ii the region of the white dwarfs **[1]**

iii the region of the red giants **[1]**

iv the region of the Cepheids. **[1]**

b Use the diagram to estimate the following ratios of radii:

i $\dfrac{R_X}{R_Y}$ **[2]**

ii $\dfrac{R_Z}{R_X}$. **[2]**

c Estimate the mass of star X. **[2]**

d **i** Show the evolutionary path of star X on the HR diagram from the main sequence until its final equilibrium state. **[1]**

ii Explain how this star remains in equilibrium in its final state. **[2]**

iii State the condition on the mass of star X in its equilibrium state. **[1]**

6 **a** State **Hubble's law**. **[1]**

Light from distant galaxies arrives on Earth red-shifted.

b Explain what **red-shifted** means. **[2]**

c Describe the origin of this red-shift. **[2]**

d Light from a distant galaxy is emitted at a wavelength of 656 nm and is observed on Earth at a wavelength of 780 nm. The distance to the galaxy is 920 Mpc.

i Calculate the velocity of recession of this galaxy. **[2]**

ii Estimate the size of the universe when the light was emitted relative to its present size. **[2]**

iii Estimate the age of the universe based on the data of this problem. **[2]**

e Outline, by reference to Type Ia supernovae, how the accelerated rate of expansion of the universe was discovered. **[3]**

7 **a** Outline how the CMB provides evidence for the Big Bang model of the universe. **[3]**

b The photons observed today in the CMB were emitted at a time when the temperature of the universe was about 3.0×10^3 K.

HL **i** Calculate the red-shift experienced by these photons from when they were emitted to the present time. **[2]**

ii Estimate the size of the universe when these photons were emitted relative to the size of the universe now. **[1]**

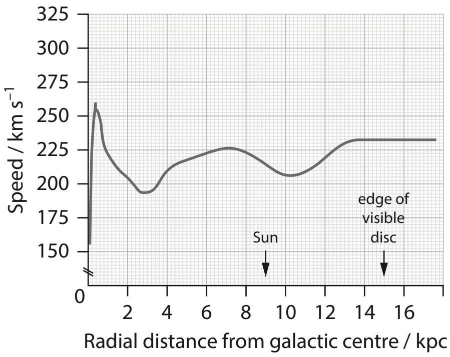
HL 8 **a** Show that the rotational speed v of a particle of mass m that orbits a central mass M at an orbital radius r is given by $v = \sqrt{\frac{GM}{r}}$. **[1]**

b Using the result in **a**, show that if the mass M is instead an extended cloud of gas with a mass distribution $M = kr$, where k is a constant, then v is constant. **[2]**

c The rotation curve of our galaxy is given graph below.

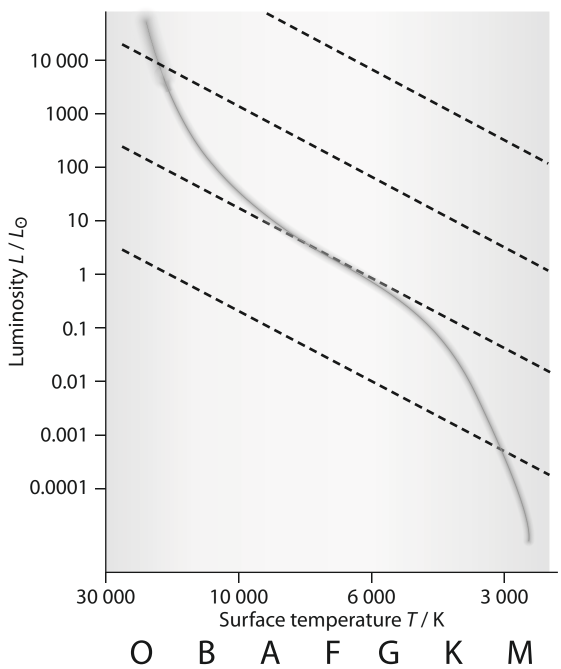
i Explain how this graph may be used to deduce the existence of dark matter. **[3]**

ii State **two** candidates for dark matter. **[2]**



HL 9 **a** Describe what is meant by the **Jeans criterion**.

b On a copy of the HR diagram below, draw the path of a protostar of mass equal to one solar mass.



c The Jeans criterion may be expressed mathematically as $\frac{GM^2}{R} \approx \frac{3}{2}NkT$, where M and R are the mass and radius of a dust cloud, N is the number of particles in the cloud and T is its temperature.

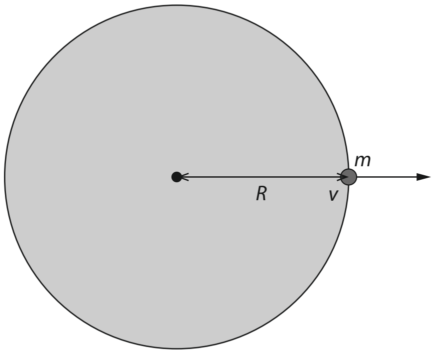
i Show that this is equivalent to the condition

$$R^2 \approx \frac{9kT}{8\pi G\rho m}$$

where m is the mass of a particle in the cloud of dust and ρ is the density of the cloud. **[3]**

ii Estimate the linear size R of a cloud that can collapse to form a protostar, assuming $T = 100\,\text{K}$, $\rho = 1.8 \times 10^{-19}\,\text{kg}\,\text{m}^{-3}$ and $m = 2.0 \times 10^{-27}\,\text{kg}$. **[2]**

HL **10** **a** The diagram below shows a particle of mass m and a spherical cloud of density ρ and radius R.



i State the speed of the particle relative to an observer at the centre of the cloud, assuming that Hubble's law applies to this particle. **[1]**

ii Show that the total energy of the particle–cloud system is

$$E = \frac{1}{2}mR^2\left(H_0^2 - \frac{8\pi\rho G}{3}\right)$$ **[2]**

iii Hence deduce that the minimum density of the cloud for which the particle can escape to infinity is

$$\rho = \frac{3H_0^2}{8\pi G}$$ **[2]**

iv Evaluate this density using $H_0 = 68\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$.

b The density found in **a iii** is known as the critical density. In the context of cosmological models and by reference to flat models of the universe, outline the significance of the critical density. **[2]**

c Current data suggest that the density of matter in our universe is 32% of the critical density.

i Calculate the matter density in our universe. **[2]**

ii For this value of the matter density, draw a sketch graph to show the variation with time of the scale factor of the universe for a model with no dark energy, and also for a model with dark energy. **[2]**

HL **11** **a** Outline what is meant by **fluctuations in the CMB**. **[2]**

b Give **two** reasons why these fluctuations are significant. **[4]**

HL **12** **a** Explain why only elements up to iron are produced in the cores of stars. **[2]**

b Outline how elements heavier than iron are produced in the course of stellar evolution. **[4]**

c Suggest why the CNO cycle takes place only in massive main-sequence stars. **[2]**