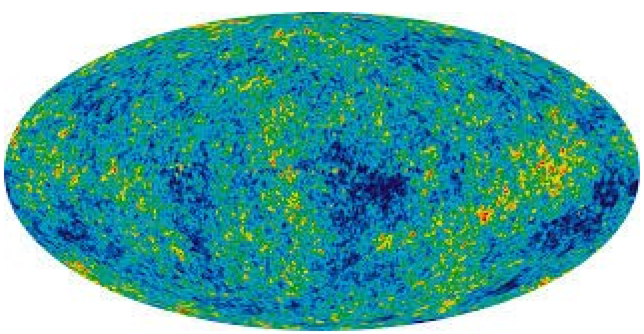
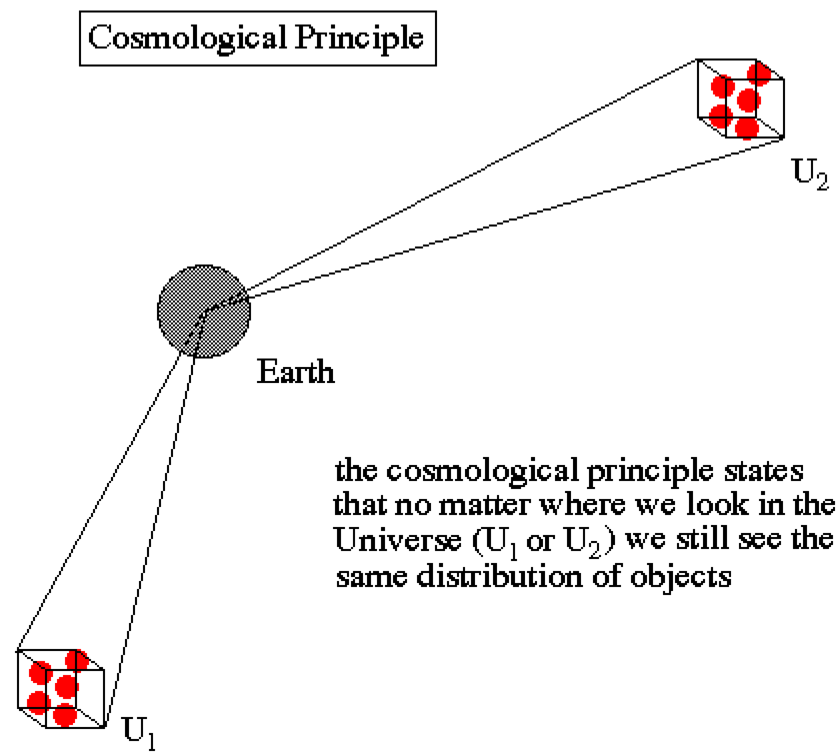
Option D5

Further cosmology

Cosmological principle

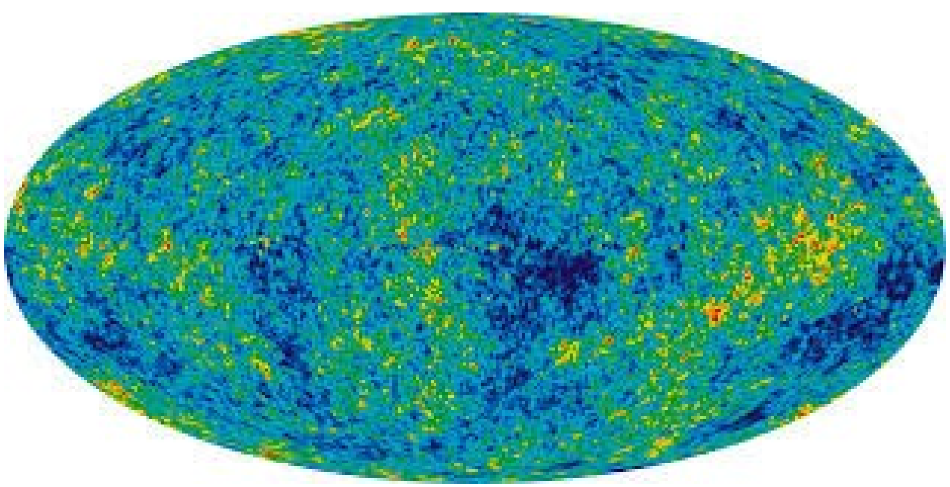
- The universe is uniform on a large scale: homogeneity principle
- It also looks the same in all directions: isotropy principle





Fluctuations in the CMB

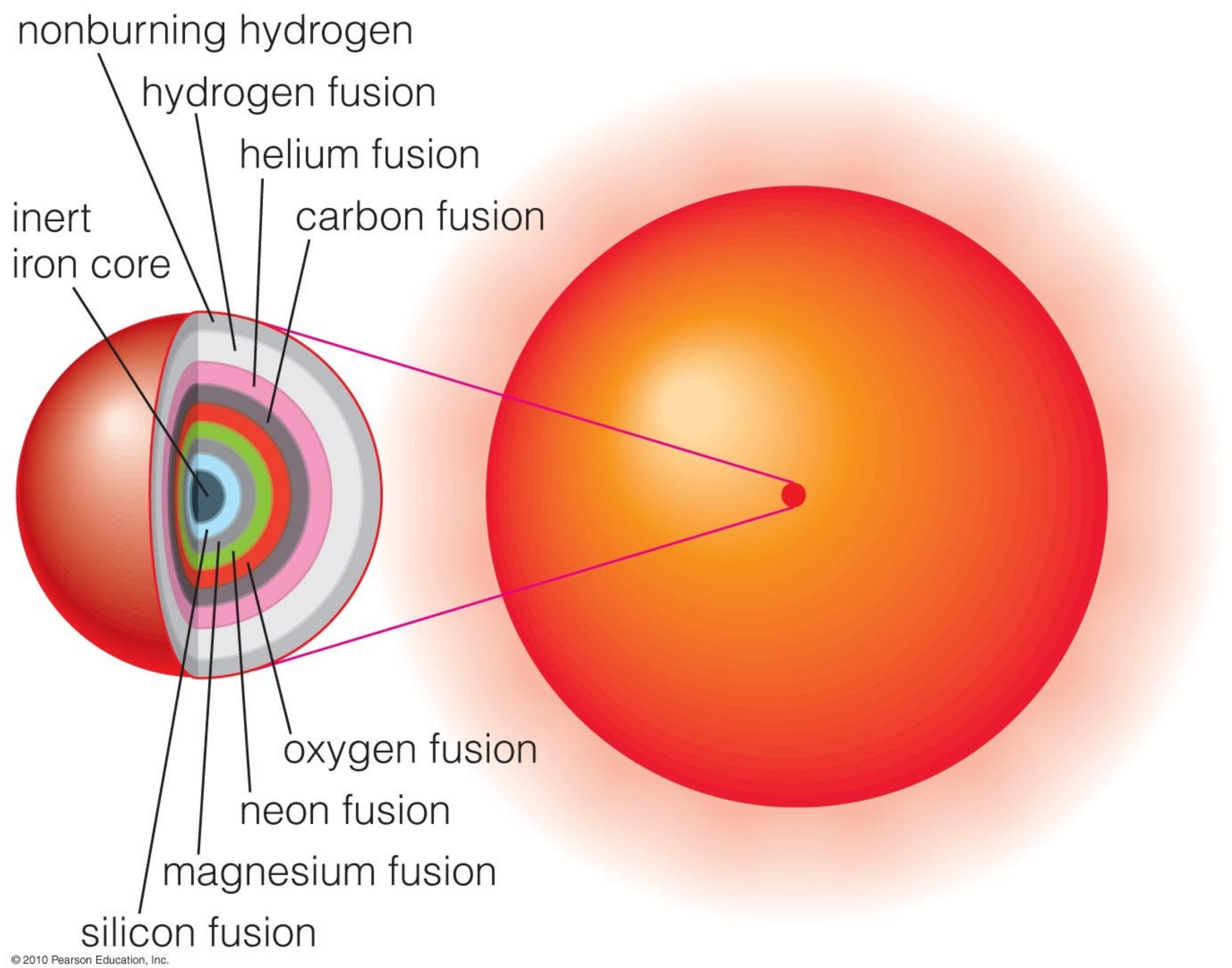
- The microwave background shows fluctuations on the order of 10^{-5}
- These fluctuations gave rise to structures that we see now as galaxy clusters
- We can also use these anisotropies to try to find the curvature of the universe: positive, zero, or negative



After main sequence?

$$^{4}He + {}^{4}He \rightleftharpoons {}^{8}Be$$
$$^{8}Be + {}^{4}He \rightarrow {}^{12}C + \gamma$$

- Triple alpha process
- This continues to iron for a high mass star



Mass luminosity relationship

$$L \propto M^{3.5}$$

- Ex: Rigel is 18 M_s. What is its luminosity?

$$L \propto 18^{3.5} = 25000$$

Mass luminosity relationship

- Ex: What is the mass of a red dwarf with 10^{-4} Ls?
- Could Jupiter be a red dwarf?

$$L \propto M^{3.5}$$

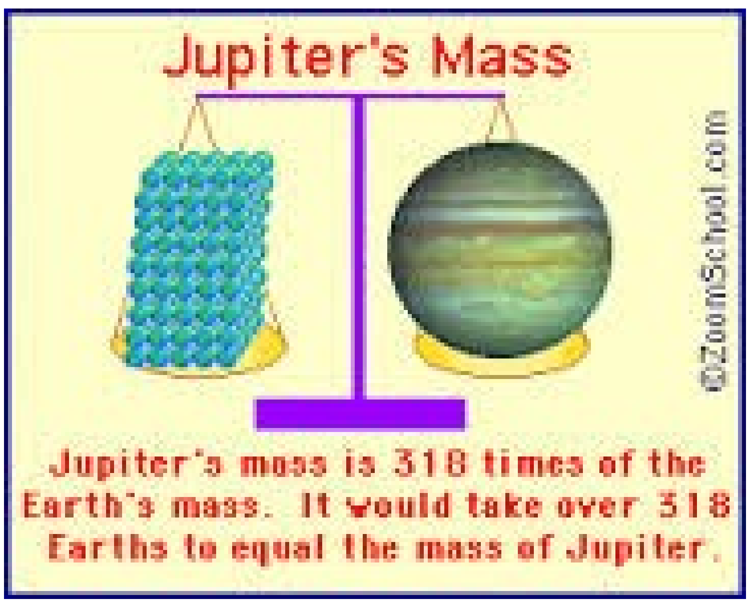
$$M \propto L^{1/3.5}$$

$$M \propto \left(10^{-4}\right)^{1/3.5} = 0.072$$

$$M = 0.072 \times 1.98 \times 10^{30}\, kg$$

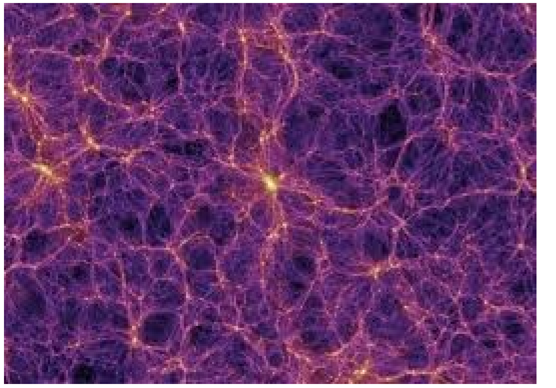
- How many Jupiters to make a red dwarf?

$$\frac{M}{M} = \frac{1.4\times10^{29}\,kg}{1.9\times10^{27}\,kg}$$



$$75\,Jupiters$$

Why dark matter?



- Consider orbital velocity:

$$v = \sqrt{\frac{GM}{r}}$$

- What about a star system orbiting around the Mass of a spherical galaxy at radius r?

$$M = \rho V = \rho \frac{4}{3}\pi r^3$$

$$v = \sqrt{\frac{G\rho \frac{4}{3}\pi r^3}{r}} = \sqrt{\frac{4\pi G\rho}{3}} r$$

- If the milky way has a 100 billion stars with an average mass of 0.3 solar masses, how fast should we be orbiting at our radius of 26000 light years? Use Sun=$1.98\text{x}10^{30}$ kg

$$v=\sqrt{\frac{GM}{r}}=\sqrt{\frac{6.67\times10^{-11}\cdot1\times10^{11}\cdot0.3\cdot1.98\times10^{30}}{26000\cdot9.46\times10^{15}}}$$

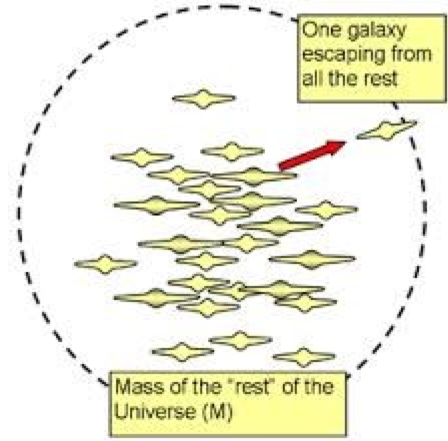
$$=1.3\times10^{5}\,m\cdot s^{-1}$$

- Actual speed?

$$v=2.0\times10^{5}\,m\cdot s^{-1}$$

- Since this is too fast for this mass to keep us in orbit, astronomers reasoned there must be extra “dark matter” providing this extra gravity

Can we solve for critical density?



- Is there enough mass to halt the expansion of the universe?

$$E = E_k + E_p$$

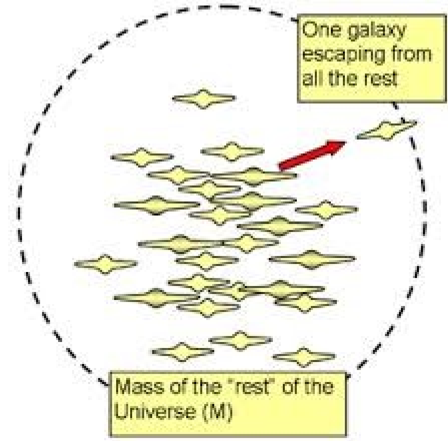
- A galaxy with mass m has energy:

$$E = \frac{1}{2}mv^2 + \frac{-GMm}{r}$$

- If we have density ρ and Hubble's Ho:

$$v = H_0 d \qquad M = \rho\frac{4}{3}\pi r^3$$

$$E = \frac{1}{2} m (H_0 d)^2 - G\rho \frac{4}{3} \pi r^3 m / r$$



$$E = \frac{1}{2} m r^2 \left(H_0{}^2 - \frac{8\pi\rho G}{3} \right)^2$$

- The critical case is for E=0 giving:

$$G\rho \frac{4}{3} \pi r^2 m = \frac{1}{2} m (H_0 d)^2$$

$$\rho = \frac{3(H_0 d)^2}{8\pi r^2 G} = \frac{3H_0{}^2}{8\pi G} \cong 10^{-26} kg \cdot m^{-3}$$

What is the fate of the universe?

- Choose one of the following to present:
  - Dark matter
  - Dark energy
  - Curvature of the universe