54 Explain why the inverse of the Hubble constant, $\frac{1}{H}$, is taken to be an estimate of the 'age of the universe'. Estimate how old the universe would be if $H = 500\,\text{km s}^{-1}\,\text{Mpc}^{-1}$ (close to Hubble's original value).

55 Explain why the Hubble constant sets an **upper bound** on the age of the universe.

56 Explain why Hubble's law does not imply that the Earth is at the centre of the universe.

57 The temperature of the cosmic microwave background radiation as measured from the Earth is about 2.7 K.

a What is the significance of this radiation?

b What would be the temperature of the CMB radiation as measured by an observer in the Andromeda galaxy, 2.5 million light years away?

58 **a** Draw a sketch graph to show the variation of the CMB radiation intensity with wavelength.

b Calculate the peak wavelength corresponding to a CMB radiation temperature of 2.72 K.

59 Predict what will happen to the temperature of the CMB radiation if:

a the universe keeps expanding forever

b the universe starts to collapse.

60 **a** State what is meant by **red-shift**.

b Describe the mechanism by which the observed red-shift in light from distant galaxies is formed.

c Show that the distance d of a galaxy with a red-shift of z is given by $d = \frac{cz}{H_0}$.

d Calculate the distance of a galaxy whose red-shift is 0.18, using $H_0 = 68\,\text{km s}^{-1}\,\text{Mpc}^{-1}$.

e Estimate the size of the universe now, relative to its size when the light in **d** was emitted.

61 The wavelength of a particular spectral line measured in the laboratory is 486 nm. The same line observed in the spectrum of a distant galaxy is shifted by 15 nm.

a Estimate the distance of the galaxy, using $H_0 = 68\,\text{km s}^{-1}\,\text{Mpc}^{-1}$.

b Estimate the size of the universe now, relative to its size when the light in **a** was emitted.

62 A photon is emitted at a time when the size of the universe was 85% of its present size. Estimate the distance from the Earth of the point from which the photon was emitted, using $H_0 = 68\,\text{km s}^{-1}\,\text{Mpc}^{-1}$.

63 State the property of Type Ia supernovae that is significant in distance measurements in cosmology.

64 **a** State what is meant by the **accelerating universe**.

b Suggest why the universe was expected to decelerate rather than accelerate.

c Outline how Type Ia supernovae were used to discover the acceleration of the universe.

d Explain why it is important to observe such a supernova starting before it reaches its peak luminosity.

65 It is said that distant supernovae appear dimmer than they would in a decelerating universe. Explain this statement.

66 It was stated in the text that Type Ia supernovae are very rare (a few in a galaxy every thousand years). Suggest how two research groups were able to observe over 50 such supernovae in the space of just a few years.

Learning objectives

- Understand and apply the Jeans criterion.
- Describe nuclear fusion in stars.
- Describe nucleosynthesis off the main sequence.
- Distinguish and describe Type Ia and Type II supernovae.

D4 Stellar processes (HL)

This section deals with the birth, evolution and death of stars, and with their role in **nucleosynthesis,** the production of elements through fusion and neutron absorption. The section closes with a discussion of supernovae and the role of Type Ia supernovae as standard candles.

D4.1 The Jeans criterion

Interstellar space (the space between stars) consists of gas and dust at a density of about $10^{-21}\,\text{kg m}^{-3}$. This amounts to about one atom of hydrogen in every cubic centimetre of space. The gas is mainly hydrogen (about 74% by mass) and helium (25%), with other elements making up the remaining 1%. Whenever the gravitational energy of a given mass of gas exceeds the average kinetic energy of the random thermal motion of its molecules, the gas becomes unstable and tends to collapse:

$$\frac{GM^2}{R} \geq \frac{3}{2}NkT$$

where k isBoltzmann's constant, T is temperature and N is the number of particles. R is the radius of the gas cloud and M its mass. This is known as the **Jeans criterion**.

Stars formed (and continue to be formed) when rather cool gas clouds in the interstellar medium ($T \approx 10$–$100\,\text{K}$) of sufficiently large mass (large enough to satisfy the Jeans criterion) collapsed under their own gravitation. In the process of contraction, the gas heated up. Typically, the collapsing gas would break up into smaller clouds, resulting in the creation of more than one star. When the temperature rises sufficiently for visible light to be emitted, a star so formed is called a **protostar** (see Figure **D.29**).

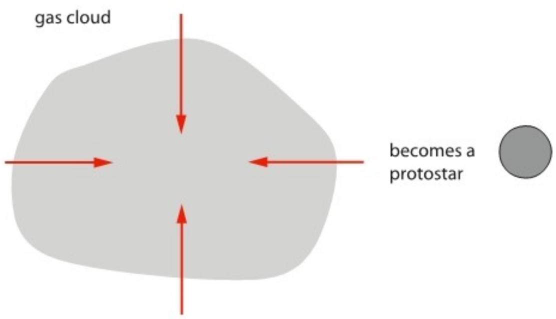

Figure D.29 The formation of a protostar out of a collapsing cloud of gas.

Worked examples

D.19 Show that the Jeans criterion can be rewritten as $M^2 = \frac{3}{4\pi\rho}\left(\frac{3kT}{2mG}\right)^3$, where ρ is the density of the gas and m is the mass of a particle of the gas. (The right hand side is known as the square of the Jeans mass.)

Cube each side of the Jeans criterion equation to find

$$\left(\frac{GM^2}{R}\right)^3 = \left(\frac{3kMT}{2m}\right)^3 \quad \Rightarrow \quad M^2 = \left(\frac{3}{4\pi\rho}\right)\left(\frac{3kT}{2mG}\right)^3$$

using the definition of density and $M = Nm$, where m is the mass of one molecule.

D.20 Take the density of interstellar gas in a cloud to be about 100 atoms of hydrogen per cm^3. Estimate the smallest mass this cloud can have for it to become unstable and begin to collapse when $T = 100\,\text{K}$.

The density is

$$\frac{100 \times 1.67 \times 10^{-27}\,\text{kg}}{10^{-6}\,\text{m}^3} = 1.67 \times 10^{-19}\,\text{kg}\,\text{m}^{-3}$$

With $T = 100\,\text{K}$ in the Jeans criterion we find (see Worked example **D.19**)

$$M \approx 3.0 \times 10^{33}\,\text{kg} = 1.5 \times 10^3\,M_\odot$$

Such a large gas cloud might well break up, forming more than one star.

D4.2 Nuclear fusion

We saw in Section **D1.2** that the main series of nuclear fusion reactions taking place in the cores of main-sequence stars is the proton–proton cycle:

$$^{1}_{1}\mathrm{H} + ^{1}_{1}\mathrm{H} \rightarrow ^{2}_{1}\mathrm{H} + ^{0}_{1}\mathrm{e}^{+} + \nu$$

$$^{1}_{1}\mathrm{H} + ^{2}_{1}\mathrm{H} \rightarrow ^{3}_{2}\mathrm{He} + \gamma$$

$$^{3}_{2}\mathrm{He} + ^{3}_{2}\mathrm{He} \rightarrow ^{4}_{2}\mathrm{He} + 2^{1}_{1}\mathrm{H}$$

In this cycle, the net effect is to turn four hydrogen nuclei into one helium nucleus.

For stars more massive than our Sun, there is a second way to fuse hydrogen into helium. This is the so-called **CNO cycle**, described by the following series of fusion reactions:

$$^{1}_{1}\mathrm{H} + ^{12}_{6}\mathrm{C} \rightarrow ^{13}_{7}\mathrm{N} + \gamma$$

$$^{13}_{7}\mathrm{N} \rightarrow ^{13}_{6}\mathrm{C} + ^{0}_{1}\mathrm{e}^{+} + \nu$$

$$^{1}_{1}\mathrm{H} + ^{13}_{6}\mathrm{C} \rightarrow ^{14}_{7}\mathrm{N} + \gamma$$

$$^{1}_{1}\mathrm{H} + ^{14}_{7}\mathrm{N} \rightarrow ^{15}_{8}\mathrm{O} + \gamma$$

$$^{15}_{8}\mathrm{O} \rightarrow ^{15}_{7}\mathrm{N} + ^{0}_{1}\mathrm{e}^{+} + \nu$$

$$^{1}_{1}\mathrm{H} + ^{15}_{7}\mathrm{N} \rightarrow ^{12}_{6}\mathrm{C} + ^{4}_{2}\mathrm{He}$$

Exam tip
The CNO cycle applies to more massive main-sequence stars and does not produce elements heavier than helium.

Notice that the net effect is to turn four hydrogen nuclei into one helium nucleus, just like the proton–proton cycle; the heavier elements produced in intermediate stages are all used up. The carbon nucleus has a charge of +6, so the barrier that must be overcome for carbon to fuse is much higher. This requires higher temperatures.

The proton–proton cycle and the CNO cycle are both main-sequence star processes. What happens beyond the main sequence?

Exam tip
The triple alpha process is how we end up with white dwarfs with a carbon core.

The first element to be produced as a star leaves the main sequence and enters the red giant stage is carbon, through the **triple alpha process**:

$$^{4}_{2}\mathrm{He} + ^{4}_{2}\mathrm{He} \rightarrow ^{8}_{4}\mathrm{Be} + \gamma$$

$$^{4}_{2}\mathrm{He} + ^{8}_{4}\mathrm{Be} \rightarrow ^{12}_{6}\mathrm{C} + \gamma$$

This happens to stars with masses up to eight solar masses. The star will shed most of its mass in a planetary nebula and end up as a white dwarf with a core of carbon.

For stars even more massive than this, helium fuses with carbon to produce oxygen:

$$^{4}_{2}\mathrm{He} + ^{12}_{6}\mathrm{C} \rightarrow ^{16}_{8}\mathrm{O} + \gamma$$

In even more massive stars, neon, sodium and magnesium are produced:

$$^{12}_{6}\mathrm{C} + ^{12}_{6}\mathrm{C} \rightarrow ^{20}_{10}\mathrm{Ne} + ^{4}_{2}\mathrm{He}$$

$$^{12}_{6}\mathrm{C} + ^{12}_{6}\mathrm{C} \rightarrow ^{23}_{11}\mathrm{Na} + ^{1}_{1}\mathrm{H}$$

$$^{12}_{6}\mathrm{C} + ^{12}_{6}\mathrm{C} \rightarrow ^{23}_{12}\mathrm{Mg} + ^{1}_{0}\mathrm{n}$$

Silicon is then produced by the fusion of oxygen:

$${}^{16}_{8}O + {}^{16}_{8}O \rightarrow {}^{28}_{14}Si + {}^{4}_{2}He$$

The process continues until iron is formed. This creates an onion-like layered structure in the star, with progressively heavier elements as we move in towards the centre. Fusion cannot produce elements heavier than iron, since the binding energy per nucleon peaks near iron and further fusion is not energetically possible. Thus, a massive star ends its cycle of nuclear reactions with iron at its core, surrounded by progressively lighter elements, as shown in Figure **D.30**.

Exam tip
You should be able to explain why fusion ends with the production of iron.

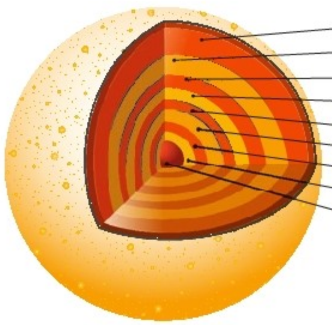

Figure D.30 The central core of a fully evolved massive star consists of iron with layers of lighter elements surrounding it.

Worked example

D.21 Our Sun emits energy at a rate (luminosity) of about 3.9×10^{26} W. Estimate the mass of hydrogen that undergoes fusion in one year. Assuming that the energy loss is maintained at this rate, find the time required for the Sun to convert 12% of its hydrogen into helium. (Mass of Sun = 1.99×10^{30} kg.)

Assuming the proton–proton cycle as the reaction releasing energy (by hydrogen fusion), the energy released per reaction is about 3.98×10^{-12} J. Since the luminosity of the Sun is 3.9×10^{26} W, it follows that the number of fusion reactions required per second is

$$\frac{3.9 \times 10^{26}}{3.98 \times 10^{-12}} = 9.8 \times 10^{37}$$

For every such reaction, four hydrogen nuclei fuse to helium and thus the mass of consumed hydrogen is $9.8 \times 10^{37} \times 4 \times 1.67 \times 10^{-27}\,\text{kg s}^{-1} = 6.5 \times 10^{11}\,\text{kg s}^{-1} = 2 \times 10^{19}$ kg per year. At the time of its creation, the Sun consisted of 75% hydrogen, corresponding to a mass of $0.75 \times 1.99 \times 10^{30}\,\text{kg} = 1.5 \times 10^{30}$ kg. The limit of 12% results in a hydrogen mass to be fused of 1.8×10^{29} kg. The time for this mass to fuse is thus

$$\frac{1.8 \times 10^{29}}{6.5 \times 10^{11}}\,\text{s} = 2.8 \times 10^{17}\,\text{s}$$
$$= 8.9 \times 10^{9}\,\text{yr}$$

Since the Sun has existed for about 5 billion years, it still has about 4 billion years left in its life as a main-sequence star.

One application of the mass–luminosity relation is to estimate the lifetime of a star on the main sequence. Since the luminosity is the power radiated by the star, we may write that

$$\frac{E}{T} \propto M^{3.5}$$

where E is the total energy radiated by the star and T is the time in which this happens. For the purposes of an estimate, we may assume that the total energy that the star can radiate comes from converting *all* its mass into energy according to Einstein's formula, $E = Mc^2$. Thus

$$\frac{E}{T} \propto M^{3.5} \quad \Rightarrow \quad \frac{Mc^2}{T} \propto M^{3.5} \Rightarrow T \propto M^{-2.5}$$

This means that the lifetimes of two stars are approximately related by

$$\frac{T_1}{T_2} = \left(\frac{M_2}{M_1}\right)^{2.5}$$

Worked example

D.22 Our Sun will spend about 10^{10} yr on the main sequence. Estimate the time spent on the main sequence by a star whose mass is 10 times the mass of the Sun.

We know that $\frac{T_1}{T_2} = \left(\frac{M_2}{M_1}\right)^{2.5}$. Hence, $\frac{T}{T_\odot} = \left(\frac{M_\odot}{10M_\odot}\right)^{2.5} \quad \Rightarrow \quad T = \frac{10^{10}}{10^{2.5}} \approx 3 \times 10^7$ yr.

D4.3 Nucleosynthesis of the heavy elements

All of the hydrogen and most of the helium in the universe were produced at the very earliest moments in the life of the universe. Everything else was made in stars in the course of stellar evolution. In Section **D4.2** we learned that nuclear fusion reactions in stellar cores produce the elements up to iron. So how are the rest of the elements in the periodic table produced?

The answer lies in neutron absorption by nuclei – **neutron capture**. When a nucleus absorbs a neutron it becomes an isotope of the original nucleus. This isotope is usually unstable and will decay. The issue is whether there is enough time for this decay to occur before the isotope absorbs yet another neutron. In what is referred to as an **s-process** (s for 'slow'), the isotope does have time to decay because the number of neutrons present is small. This happens in stars where relatively small numbers of neutrons are produced in the fusion reactions discussed in Section **D4.2**. The isotope will undergo a series of decays, including beta decay, in which the atomic number is increased by one, thus producing a new element. This process accounts for the production of about half of the nuclei above iron, and ends with the production of bismuth 209.

By contrast, in the presence of very large numbers of neutrons, nuclei that absorb neutrons do not have time to decay. In an **r-process** (r for 'rapid'), they keep absorbing neutrons one by one, forming very heavy, neutron-rich isotopes. This cannot happen inside a star but it does happen during a supernova explosion. These neutron-rich isotopes are then hurled into space by the supernova, where they can undergo beta decay, producing nuclei of higher atomic number.

Beta decay is not the only way to turn a neutron into a proton and hence increase the atomic number. In supernova explosions, massive numbers of neutrinos are produced. A neutron may absorb a neutrino and turn into a proton according to the reaction

$${}^{1}_{0}n + \nu \rightarrow {}^{1}_{1}p + {}^{0}_{-1}e$$

D4.4 Type Ia and Type II supernovae

The supernovae we learned about in Section **D2.8** referred to the explosion of red supergiant stars. These are called **Type II supernovae**. Another type of supernovae, called **Type Ia supernovae**, involve a different mechanism.

Consider a **binary star** system in which one of the stars is a white dwarf. This star may attract material from its companion star. Mass falling into the white dwarf may increase the white dwarf's mass beyond the Chandrasekhar limit, so nuclear fusion reactions may start again in its core. The resulting sudden release of energy appears as a sudden increase in the luminosity of the white dwarf – that is, a supernova. Unlike Type II supernovae, Type Ia supernovae show no hydrogen lines.

Figure **D.31** shows the remnants of two supernovae.

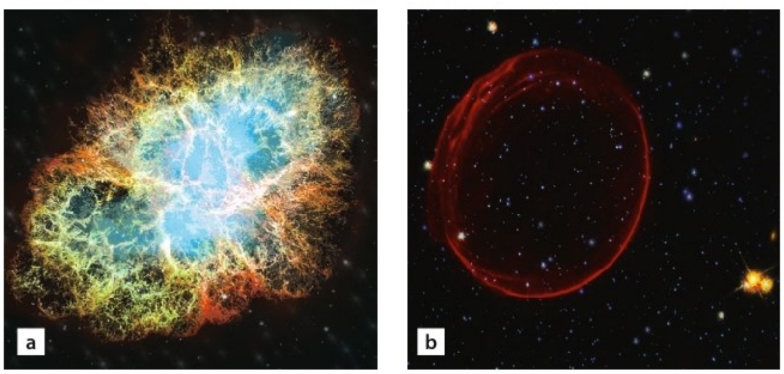

Figure D.31 **a** The Crab nebula, the remnant of a Type II supernova observed by the Chinese in 1054 and possibly by Native Americans. It was visible for weeks, even during daytime. **b** This majestically serene bubble of gas in space is the remnant of the Type Ia supernova SNR 0509, which exploded about 400 years ago.

Exam tip

Differences in the two types of supernovae:

Type Ia

- do not have hydrogen lines in their spectra
- are produced when mass from a companion star accretes onto a white dwarf, forcing it to exceed the Chandrasekhar limit
- have a luminosity which falls off sharply after the explosion.

Type II

- have hydrogen lines in their spectra
- are produced when a massive red supergiant star explodes
- have a luminosity which falls off gently after the explosion.

As noted in Section **D3.5**, an important property of Type Ia supernovae is that they all have the same peak luminosity and so may be used as 'standard candles'. By measuring their apparent brightness at the peak, we may calculate their distance from $d = \sqrt{\frac{L}{4\pi b}}$ (see Section **D1.5**).

Apart from the mechanism producing them, the two types of supernovae also differ in that Type Ia supernovae do not have hydrogen absorption lines in their spectra, whereas Type II do. They also differ in the way the luminosity falls off with time (Figure **D.32**).

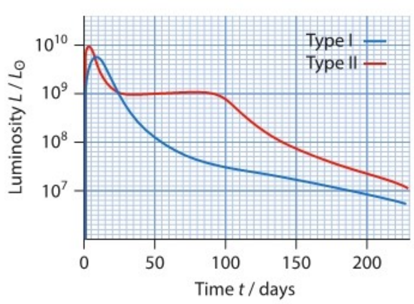

Figure D.32 Type Ia and Type II supernovae show different graphs of luminosity versus time.

Nature of science

Observation and deduction

Stellar spectra show us the elements present in stars' outer layers, but how do we explain how these elements came to be formed? Astrophysicists have shown how nuclear fusion reactions in the stars produce energy and also build up the elements. Modelling and computer simulations have resulted in a picture that agrees very well with observation. The time scales of stellar processes are much too long for us to directly observe the phenomena of stellar evolution, but the presence of stars of different ages and at different stages of evolution allows a comparison of observations and theoretical predictions.

Test yourself

67 Describe the **Jeans criterion**.

68 Describe how a gas and dust cloud becomes a protostar.

69 Explain whether star formation is more likely to take place in cold or hot regions of interstellar space.

70 Explain why a star at the top left of the main sequence will spend much less time on the main sequence than a star at the lower right.

71 Show that a star that is twice as massive as the Sun has a lifetime that is 5.7 times shorter than that of the Sun.

72 Using the known luminosity of the Sun and assuming that it stays constant during the Sun's lifetime, which is estimated to be 10^{10} yr, calculate the mass this energy corresponds to according to Einstein's mass–energy formula.

73 Suggest why the depletion of hydrogen in a star is such a significant event.

74 Evolved stars that have left the main sequence have an onion-like layered structure. Outline how this structure is created.

75 Describe the nuclear reactions taking place in a star of one solar mass:
- **a** while the star is on the main sequence
- **b** after it has left the main sequence.

76 Describe the nuclear reactions taking place in a star of 20 solar masses:
- **a** while on the main sequence
- **b** after it has left the main sequence.

77 Explain why no elements heavier than iron are produced in stellar cores.

78 State the element that is the end product of:
- **a** the proton–proton cycle
- **b** the CNO cycle
- **c** the triple alpha process.

79 Distinguish between an **s-process** and an **r-process**.

80 Suggest why the production of heavier elements inside stars requires higher temperatures.

81 Describe how a Type Ia supernova is formed.

82 Describe how a Type II supernova is formed.

83 State **three** differences between Type Ia and Type II supernovae.

84 Suggest why hydrogen lines are expected in the spectra of Type II supernovae.

85 Compare and contrast the proton–proton and CNO cycles.