## Learning objectives

- Understand Hubble's law.
- Understand the scale factor and red-shift.
- Understand the cosmic microwave background radiation.
- Understand the accelerating universe and red-shift.

# D3 Cosmology

This section deals with three dramatic discoveries in cosmology: the discovery of the expansion of the universe by Hubble, the discovery of the **cosmic background radiation** by Penzias and Wilson, and finally the discovery of the accelerated rate of expansion by Perlmutter, Schmidt and Riess.

## D3.1 Hubble's law: the expanding universe

Early in the 20th century, studies of galaxies revealed red-shifted absorption lines. Application of the standard Doppler effect indicated that these galaxies were moving **away from us**. By 1925, 45 galaxies had been studied, and all but the closest ones appeared to be moving away at enormous speeds.

### Physics in distant galaxies is the same as that on Earth

How do we know the wavelength of light emitted by distant galaxies? Light emitted from galaxies comes from atomic transitions in the hot gas in the interior of the galaxies, which is mostly hydrogen. Galaxies are surrounded by cooler gas and thus light travelling through is absorbed at specific wavelengths, showing a characteristic absorption spectrum. The wavelengths corresponding to the dark lines are well known from experiments on Earth.

The velocity of recession is found by an application of the Doppler effect to light. Light from galaxies arrives on Earth red-shifted. This means that the wavelength of the light measured upon arrival is longer than the wavelength at emission. According to the Doppler effect, this implies that the source of the light – the galaxy – is moving away from observers on Earth.

The Doppler effect *may* be used to describe the red-shift in the light from distant galaxies. However, the red-shift is a consequence of the expanding universe in the sense that the space between galaxies is stretching out (expands) and this gives the illusion of galaxies moving away from each other; see Section **D3.2**.

If $\lambda_0$ is the wavelength of a spectral line and $\lambda$ is the (longer) wavelength received on Earth, the red-shift $z$ of the galaxy is defined as

$$z = \frac{\lambda - \lambda_0}{\lambda_0}$$

If the speed $v$ of the receding galaxy is small compared with the speed of light $c$, then the Doppler formula is $z = \frac{v}{c}$, which shows that the red-shift is indeed directly proportional to the receding galaxy's speed (more correctly, the component of its velocity along the line of sight).

In 1925, Edwin Hubble began a study to measure the distance to the galaxies for which the velocities of recession had been determined. In

**Exam tip**

Notice that $z = \frac{\lambda - \lambda_0}{\lambda_0}$ can also be written as $z = \frac{\lambda}{\lambda_0} - 1$.

The proper interpretation of the red-shift is not through the Doppler effect (even though we use the Doppler formula) but the stretching of space in between galaxies as the universe expands.

1929, Hubble announced that distant galaxies move away from us with speeds that are proportional to their distance (Figure **D.24**).

Hubble studied a large number of galaxies and found that the more distant the galaxy, the faster it moves away from us. This is **Hubble's law**, which states that the velocity of recession is directly proportional to the distance, or

$$v = H_0 d$$

where $d$ is the distance between the Earth and the galaxy and $v$ is the galaxy's velocity of recession. The constant of proportionality, $H_0$, is the slope of the graph and is known as the **Hubble constant**.

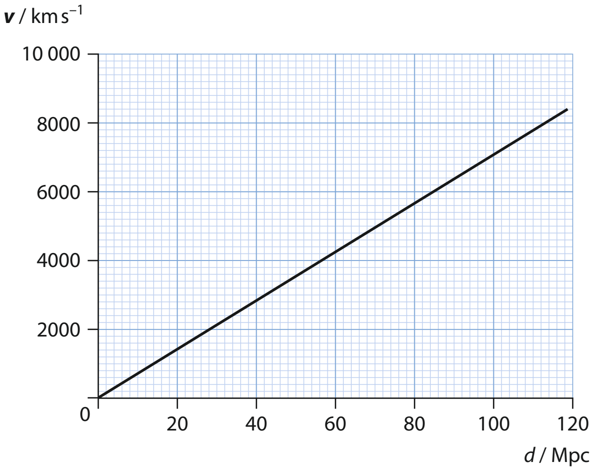



**Figure D.24** Hubble discovered that the velocity of recession of galaxies is proportional to their distance from us.

Using $z = \frac{v}{c}$, we can rewrite Hubble's law as

$$z = \frac{H_0 d}{c} \quad \Rightarrow \quad d = \frac{cz}{H_0}$$

This formula relates distance to red-shift. It is an approximate relation, valid only for values of the red-shift $z$ up to about 0.2.

There has been considerable debate as to the value of the Hubble constant. The most recent value, provided by the ESA's Planck satellite observatory data, is $H_0 = 67.80 \pm 0.77\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$.

## Worked example

**D.14** A hydrogen line has a wavelength of 434 nm. When received from a distant galaxy, this line is measured on Earth to be at 486 nm. Calculate the speed of recession of this galaxy.

The red-shift is $\frac{486 - 434}{434} = 0.12$, so $v = 0.12c = 3.6 \times 10^7\,\text{m}\,\text{s}^{-1}$.

Hubble's discovery implies that, in the past, the distances between galaxies were smaller and, moreover, that at a specific time in the past the entire universe had the size of a point. This specific time is taken to be the beginning of the universe, and leads to the model of the universe known as the **Big Bang model**, to be discussed in Section **D3.3**. Not only time, but also the space in which the matter and energy of the universe reside were created at that moment. As the space expanded, the distance between clumps of matter increased, leading to the receding galaxies that Hubble observed.

Hubble's law does not imply that the Earth is at the centre of the universe, even though the observation of galaxies moving away from us might lead us to believe so. An observer on a different star in a different galaxy would reach the same (erroneous) conclusion about their location.

## D3.2 The cosmic scale factor *R* and red-shift

The expansion of the universe can be described in terms of a scale factor, $R$. To understand what this is, consider two points with coordinates 1 and 3 on a number line (Figure **D.25a**). In ordinary geometry we would have no problem saying that the distance between the two points is the difference in their coordinates, $3-1=2$ units. Let us call this difference in coordinates $\Delta x$. If space expands, however, after some time the diagram would look like Figure **D.25b**. The distance between the points has increased but the difference in their coordinates has remained the same. So this difference does not give the actual physical distance between the points.

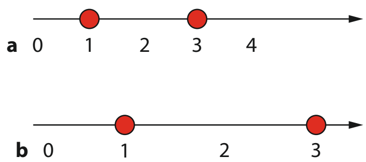



**Figure D.25** As the space between two points stretches, the physical distance between them increases, even though the coordinates of the points do not change.

The meaning of the scale factor $R$ is that multiplying the difference in coordinates $\Delta x$ by $R$ gives the physical distance $d$ between the points:

$$d = R\Delta x$$

The scale factor may depend on time. The function $R(t)$ is called the **scale factor** of the universe and is of basic importance to cosmology. It is sometimes referred to (very loosely) as the **radius of the universe**.

This gives a new and completely different interpretation of red-shift. Suppose that, when a photon of cosmic microwave background (CMB) radiation was emitted in the very distant past, its wavelength was $\lambda_0$. Let $\Delta x$ stand for the difference in the coordinates of two consecutive wave crests. Then

$$\lambda_0 = R_0\Delta x$$

where $R_0$ is the value of the scale factor at the time of emission. If this same wavelength is observed now, its value will be

$$\lambda = R\Delta x$$

where $R$ is the value of the scale factor at the present time. We deduce that

$$\frac{\lambda}{\lambda_0} = \frac{R}{R_0}$$

Thus the explanation for the observed red-shift in the light received from distant galaxies is not that the galaxies are really moving but that the space in between us and the galaxies is stretching (i.e. expanding).

**Exam tip**
SL students should know the result $z = \frac{R}{R_0} - 1$ but will not be examined on its derivation. HL students must know its derivation.

Of course, the stretching of space does give the illusion of motion, which is why applying the Doppler formula also gives the red-shift. But it must be stressed that the real explanation of the red-shift is the expansion of space and not the Doppler effect (see Figure **D.26**).

So the red-shift formula $z = \frac{\lambda - \lambda_0}{\lambda_0}$ becomes $z = \frac{R}{R_0} - 1$.

One of the great problems in cosmology is to determine how the scale factor depends on time. We will look at this problem in Section **D5.2**.

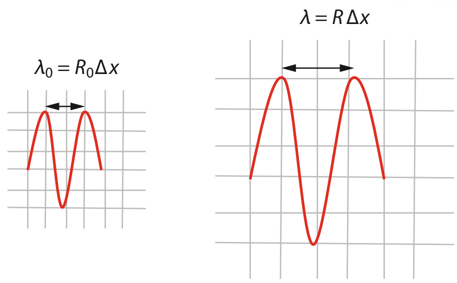



**Figure D.26** As the universe expands, the wavelength of a photon emitted in the distant past increases when measured at the present time.

## Worked examples

**D.15** The peak wavelength of the CMB radiation at present is about 1 mm. In the past there was a time when the peak wavelength corresponded to blue light (400 nm). Estimate the size of the universe then, compared with its present size.

From $\frac{\lambda}{\lambda_0} = \frac{R}{R_0}$ we find

$$\frac{R}{R_0} = \frac{1 \times 10^{-3}}{4 \times 10^{-7}} \approx 2 \times 10^3$$

So when the universe was bathed in blue light it was smaller by a factor of about 2000.

**D.16** Determine the size of the universe relative to its present size at the time when the light of Worked example **D.14** was emitted.

The red-shift was calculated to be $z = 0.12$, so from $z = \frac{R}{R_0} - 1$ we find

$$0.12 = \frac{R}{R_0} - 1 \quad \Rightarrow \quad \frac{R}{R_0} = 1.12$$

The universe was 1.12 times smaller or $\frac{100\%}{1.12} = 89\%$ of its present size when the light was emitted.

## D3.3 The Hot Big Bang model: the creation of space and time

The discovery of the expanding universe by Hubble implies a definite beginning, some 13.8 billion years ago. The size of the universe at that time was infinitesimally small and the temperature was enormous. Time, space, energy and mass were created at that instant. It is estimated that just $10^{-44}$ s after the beginning (the closest to $t = 0$ about which something remotely reliable may be said), the temperature was of order $10^{32}$ K. These conditions create the picture of a gigantic explosion at $t = 0$, which set matter moving outwards. Billions of years later, we see the remnants of this explosion in the receding motion of the distant galaxies. This is known as the **Hot Big Bang** scenario in cosmology.

The Big Bang was not an explosion that took place at a specific time in the past somewhere in the universe. At the time of the Big Bang, the

### Why is the night sky dark?

The astronomers de Cheseaux and Olbers asked the very simple question of why the night sky is dark. Their argument, based on the prevailing static and infinite cosmology of the period, led to a night sky that would be uniformly bright! In its simplest form, the argument says that no matter where you look you will end up with a star. Hence the night sky should be uniformly bright, which it is not. This is Olbers' Paradox.

space in which the matter of the universe resides was created as well. Thus, the Big Bang happened about 13.8 billion years ago everywhere in the universe (the universe then being a point).

It is important to understand that the universe is not expanding into empty space. The expansion of the universe is not supposed to be like an expanding cloud of smoke that fills more and more volume in a room. The galaxies that are moving away from us are not moving into another, previously unoccupied, part of the universe. *Space is being stretched* in between the galaxies and so the distance between them is increasing, creating the illusion of motion of one galaxy relative to another.

There is plenty of experimental evidence in support of the Big Bang model. The first is the observation of an expanding universe that we have already talked about. The next piece of evidence is the cosmic microwave background radiation, to be discussed in Section **D3.4**.

If we assume that the expansion of the universe has been constant up to now, then $\frac{1}{H_0}$ gives an upper bound on the **age of the universe**. This is only an upper bound, since the fact that expansion rate was faster at the beginning implies a younger universe. The time $\frac{1}{H_0}$, known as the **Hubble time**, is about 14 billion years. The universe cannot be older than that. A more detailed argument that leads to this conclusion is as follows. Imagine a galaxy which is now at a distance $d$ from us. Its velocity is thus $v=H_0 d$. In the beginning the galaxy and the Earth were at zero separation from each other. If the present separation of $d$ was thus covered at the same constant velocity $H_0 d$, the time $T$ taken to achieve this separation must be given by $H_0 d=\frac{d}{T}$, that is, $T=\frac{1}{H_0}$. $T$ is thus a measure of the age of the universe.

**Exam tip**
The inverse of the Hubble constant gives an **upper bound** on the age of the universe – that is, the actual age is less. This is because the estimate is based on a constant rate of expansion equal to the present rate.

The numerical value of the Hubble time with $H_0=67.80\times10^3\,\text{m}\,\text{s}^{-1}\,\text{Mpc}^{-1}$ is

$$T_H=\frac{1}{H}$$

$$=\frac{1}{67.80\times10^3\,\text{m}\,\text{s}^{-1}\,\text{Mpc}^{-1}}=\frac{1}{67.80\times10^3\,\text{m}\,\text{s}^{-1}}\times10^6\,\text{pc}$$

$$=\frac{1}{67.80\times10^3\,\text{m}\,\text{s}^{-1}}\times10^6\times3.09\times10^{16}\,\text{m}=4.557\times10^{17}\,\text{s}$$

$$=\frac{4.557\times10^{17}\,\text{s}}{365\times24\times60\times60\,\text{s}\,\text{yr}^{-1}}=14.5\times10^9\,\text{yr}$$

This assumes that the universe has been expanding at a constant rate. This is not the case, and so this is an overestimate. The actual age of the universe according to the data from the Planck satellite is 13.8 billion years.

**Exam tip**
The characteristics of the cosmic microwave background are:
- a spectrum corresponding to black-body radiation at a temperature of 2.7 K.
- peak radiation in the microwave region.
- isotropic radiation with no apparent source.

## D3.4 The cosmic microwave background radiation

In 1964, Arno Penzias and Robert Wilson, two radio astronomers working at Bell Laboratories, made a fundamental, if accidental, discovery. They were using an antenna they had just designed to study radio signals from our galaxy. But the antenna was picking up a signal that persisted no matter what part of the sky the antenna was pointing at. The spectrum of this signal (that is, the amount of energy as a function of the wavelength) turned out to be a black-body spectrum corresponding to a temperature of 2.7 K. The **isotropy** of this radiation (the fact that it was the same in all directions) indicated that it was not coming from any particular spot in the sky; rather, it was radiation that was filling all space.

Penzias and Wilson did not know that this kind of radiation had been predicted on the basis of the Big Bang theory 30 years earlier by George Gamow and his co-workers, and more recently by Jim Peebles and Robert Dicke at Princeton. The Princeton group was in fact planning to start a search for this radiation when the news of the discovery arrived.

**Penzias and Wilson, with help from the Princeton group, realised that the radiation detected was the remnant of the hot explosion at the beginning of time. It was the afterglow of the enormous temperatures that existed in the very early universe. As the universe has expanded, the temperature has fallen to its present value of 2.7 K.**

Since the work of Penzias and Wilson, a number of satellite observatories – COBE (COsmic Background Explorer), WMAP (Wilkinson Microwave Anisotropy Probe) and the Planck satellite – have verified the black-body nature of this cosmic microwave background radiation to extraordinary precision and measured its present temperature to be 2.723 K.

The COBE collaboration was headed by John Mather, who was in charge of over a thousand scientists and engineers.

## Worked example

**D.17** Find the wavelength at which most CMB radiation is emitted.

From the Wien displacement law, $\lambda T = 2.9 \times 10^{-3}$ K m, it follows that most of the energy is emitted at a wavelength of $\lambda = 1.07$ mm, which is in the microwave region.

## D3.5 The accelerating universe and red-shift

It was expected that the rate of expansion of the universe should be slowing down. This was a reasonable expectation based on the fact that gravity should be pulling back on the distant galaxies, slowing them down. Cosmologists were very much interested in determining the value of the **deceleration parameter** of the universe, a dimensionless number called $q_0$ that would quantify the deceleration. A positive value of $q_0$ would indicate deceleration and a slowing down of the expansion rate. No one doubted that $q_0$ would be positive; the only issue was its actual value.

Two groups started work in this direction. The first group, under Saul Perlmutter, started the search in 1988 and the other, led by Brian Schmidt and Adam Riess, started in 1994.

We mentioned earlier that the distance–red-shift relation, $d = \frac{cz}{H_0}$, is only approximate, valid for $z < 0.2$. But the two groups were looking for very distant supernovae and therefore high $z$ values. A more accurate relation in this case is

$$d = \frac{cz}{H_0}\left(1 + \frac{1}{2}(1 - q_0)z\right)$$

**Exam tip**
You will not be examined on the formula
$d = \frac{cz}{H_0}\left(1 + \frac{1}{2}(1 - q_0)z\right)$
but knowing of its existence is crucial in understanding how the conclusion of an **accelerating universe** was reached.

We see that the deceleration parameter $q_0$ appears in this formula. So, *in theory*, the task looked easy: find distant objects, measure their distance $d$ and red-shift $z$ and use the data to determine $q_0$. The two groups chose to look at distant **Type Ia supernovae**. The nature of these will be discussed in more detail in Section **D4.4**.

Type Ia supernovae are very rare: only a few would be expected to occur in a galaxy every thousand years! The great discovery about Type Ia supernovae is that they all have the same peak luminosity and so may be used as standard candles. Figure **D.27** shows how the logarithm of the luminosity (in W) of a Type Ia supernova varies with time.

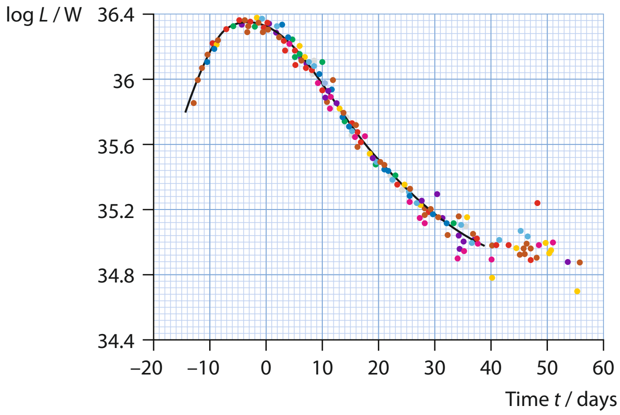



**Figure D.27** Different Type Ia supernovae all have the same peak luminosity. (Adapted from graph 'Low redshift type 1a template lightcurve', Supernova Cosmology Project/Adam Reiss)

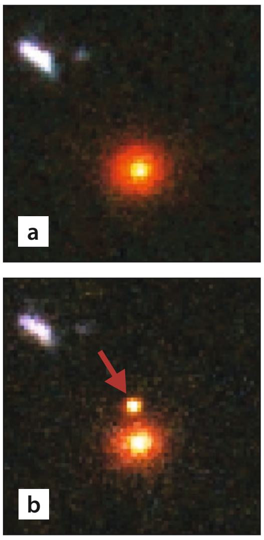



**Figure D.28** Observation of a Type Ia supernova, **a** before and **b** after outburst.

The peak luminosity is a staggering $2 \times 10^{36}$ W and falls off with time over a couple of months. If one is lucky enough to observe a Type Ia supernova from before the peak luminosity is reached, the peak apparent brightness $b$ may be measured, and since the peak luminosity $L$ is known we may find the distance to the supernova using $b = \frac{L}{4\pi d^2}$.

The task which looked easy in theory was formidable in practice, but a total of 45 supernovae were studied by the first group and 16 by the second (Figure **D.28**). Both groups were surprised to find that the deceleration parameter came out negative. This meant that the rate at which distant objects are moving away from us is **increasing**: the universe is **accelerating**. Perlmutter, Schmidt and Riess shared the 2011 Nobel Prize in Physics for this extraordinary discovery.

**Exam tip**
A 'standard candle' refers to a star of known luminosity. Thus measuring its apparent brightness gives its distance.

## Worked example

**D.18** Show that at a temperature $T = 10^{10}$ K there is enough thermal energy to create electron–positron pairs.

The thermal energy corresponding to $T = 10^{10}$ K is $E_k \approx \frac{3}{2}kT = 1.5 \times 1.38 \times 10^{-13}$ J $= 1.3$ MeV. The **rest energy** of an electron is $m_e c^2 \approx 0.5$ MeV, so the thermal energy $E_k \approx 1.3$ MeV is enough to produce a pair.

## Nature of science

### Occam's razor

Any theory of the origin of the universe must fit with the data available. The principle of **Occam's razor** says that the simplest explanation is likely to be the best. In the debate between conflicting cosmological theories, an expanding universe and the existence of a background radiation were predicted by the Big Bang theory. The red-shift in the light from distant galaxies was evidence for expansion, but it was the discovery of the cosmic microwave background radiation that led to the acceptance of the Big Bang theory. Although this theory explains many aspects of the universe as we see it now, it cannot explain what happened at the very instant the universe was created.

## Test yourself

**43** **a** State and explain **Hubble's law**.
**b** Explain how this law is evidence for an expanding universe.

**44** Some galaxies actually show a blue-shift, indicating that they are moving towards us. Discuss whether this violates Hubble's law.

**45** Galaxies are affected by the gravitational pull of neighbouring galaxies and this gives rise to what are called **peculiar** velocities. Typically these are about $500\,\mathrm{km\,s^{-1}}$. Estimate how far away a galaxy should be so that its velocity of recession due to the expanding universe equals its peculiar velocity.

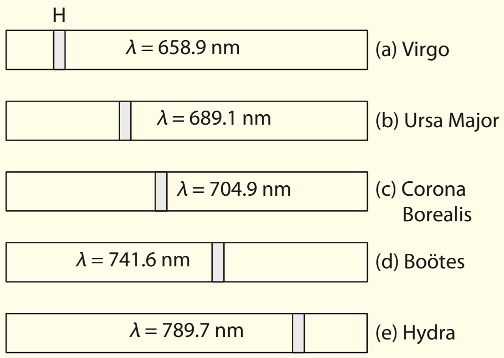
**46** A student explains the expansion of the universe as follows: 'Distant galaxies are moving at high speeds into the vast expanse of empty space.' Suggest what is wrong with this statement.

**47** It is said that the Big Bang started everywhere in space. Suggest what this means.

**48** In the context of the Big Bang theory, explain why the question 'what existed before the Big Bang?' is meaningless.

**49** Suppose that at some time in the future a detailed study of the Andromeda galaxy and all the nearby galaxies in our Local Group will be possible. Discuss whether this would help in determining Hubble's constant more accurately.

**50** The diagram shows two lines due to calcium absorption in the spectra of five galaxies, ranging from the nearby Virgo to the very distant Hydra. Each diagram gives the wavelength of the H (hydrogen) line. The wavelength of the H line in the lab is 656.3 nm. Using Hubble's law, find the distance to each galaxy. (Use $H = 68\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.)




**51** Take Hubble's constant $H$ at the present time to be $68\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.
**a** Estimate at what distance from the Earth the speed of a receding galaxy is equal to the speed of light.
**b** Suggest what happens to galaxies that are beyond this distance.
**c** The theory of special relativity states that nothing can exceed the speed of light. Suggest whether the galaxies in **b** violate relativity.

**52** Discuss **three** pieces of evidence that support the Big Bang model of the universe.

**53** A particular spectral line, when measured on Earth, corresponds to a wavelength of $4.5 \times 10^{-7}\,\mathrm{m}$. When received from a distant galaxy, the wavelength of the same line is measured to be $5.3 \times 10^{-7}\,\mathrm{m}$.
**a** Calculate the red-shift for this galaxy.
**b** Estimate the speed of this galaxy relative to the Earth.
**c** Estimate the distance of the galaxy from the Earth. (Take $H = 68\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.)

**54** Explain why the inverse of the Hubble constant, $\frac{1}{H}$, is taken to be an estimate of the 'age of the universe'. Estimate how old the universe would be if $H = 500\,\text{km s}^{-1}\,\text{Mpc}^{-1}$ (close to Hubble's original value).

**55** Explain why the Hubble constant sets an **upper bound** on the age of the universe.

**56** Explain why Hubble's law does not imply that the Earth is at the centre of the universe.

**57** The temperature of the cosmic microwave background radiation as measured from the Earth is about 2.7 K.

- **a** What is the significance of this radiation?
- **b** What would be the temperature of the CMB radiation as measured by an observer in the Andromeda galaxy, 2.5 million light years away?

**58** **a** Draw a sketch graph to show the variation of the CMB radiation intensity with wavelength.

- **b** Calculate the peak wavelength corresponding to a CMB radiation temperature of 2.72 K.

**59** Predict what will happen to the temperature of the CMB radiation if:

- **a** the universe keeps expanding forever
- **b** the universe starts to collapse.

**60** **a** State what is meant by **red-shift**.

- **b** Describe the mechanism by which the observed red-shift in light from distant galaxies is formed.
- **c** Show that the distance $d$ of a galaxy with a red-shift of $z$ is given by $d = \frac{cz}{H_0}$.
- **d** Calculate the distance of a galaxy whose red-shift is 0.18, using $H_0 = 68\,\text{km s}^{-1}\,\text{Mpc}^{-1}$.
- **e** Estimate the size of the universe now, relative to its size when the light in **d** was emitted.

**61** The wavelength of a particular spectral line measured in the laboratory is 486 nm. The same line observed in the spectrum of a distant galaxy is shifted by 15 nm.

- **a** Estimate the distance of the galaxy, using $H_0 = 68\,\text{km s}^{-1}\,\text{Mpc}^{-1}$.
- **b** Estimate the size of the universe now, relative to its size when the light in **a** was emitted.

**62** A photon is emitted at a time when the size of the universe was 85% of its present size. Estimate the distance from the Earth of the point from which the photon was emitted, using $H_0 = 68\,\text{km s}^{-1}\,\text{Mpc}^{-1}$.

**63** State the property of Type Ia supernovae that is significant in distance measurements in cosmology.

**64** **a** State what is meant by the **accelerating universe**.

- **b** Suggest why the universe was expected to decelerate rather than accelerate.
- **c** Outline how Type Ia supernovae were used to discover the acceleration of the universe.
- **d** Explain why it is important to observe such a supernova starting before it reaches its peak luminosity.

**65** It is said that distant supernovae appear dimmer than they would in a decelerating universe. Explain this statement.

**66** It was stated in the text that Type Ia supernovae are very rare (a few in a galaxy every thousand years). Suggest how two research groups were able to observe over 50 such supernovae in the space of just a few years.

### Learning objectives

- Understand and apply the Jeans criterion.
- Describe nuclear fusion in stars.
- Describe nucleosynthesis off the main sequence.
- Distinguish and describe Type Ia and Type II supernovae.

# D4 Stellar processes (HL)

This section deals with the birth, evolution and death of stars, and with their role in **nucleosynthesis,** the production of elements through fusion and neutron absorption. The section closes with a discussion of supernovae and the role of Type Ia supernovae as standard candles.

## D4.1 The Jeans criterion

Interstellar space (the space between stars) consists of gas and dust at a density of about $10^{-21}\,\text{kg m}^{-3}$. This amounts to about one atom of hydrogen in every cubic centimetre of space. The gas is mainly hydrogen (about 74% by mass) and helium (25%), with other elements making up