

Option D Astrophysics

D1 Stellar quantities

This section begins with a brief description of the various objects that comprise the universe, especially stars. We discuss astronomical distances and the main characteristics of stars: their luminosity and apparent brightness. Table D.1 presents a summary of key terms.

D1.1 Objects in the universe

We live in a part of space called the **solar system**: a collection of eight major planets (Mercury, Venus, the Earth, Mars, Jupiter, Saturn, Uranus and Neptune) bound in elliptical orbits around a star we call the Sun. Pluto has been stripped of its status as a major planet and is now called a ‘dwarf planet’. The orbit of the Earth is almost circular; that of Mercury is the most elliptical. All planets revolve around the Sun in the same direction. This is also true of the comets, with a few exceptions, the most famous being Halley’s comet. All the planets except Mercury and Venus have moons orbiting them.

Leaving the solar system behind, we enter interstellar space, the space between stars. At a distance of 4.2 light years (a light year is the distance travelled by light in one year) we find Proxima Centauri, the nearest star to us after the Sun. Many stars find themselves in **stellar clusters**, groupings of large numbers of stars that attract each other gravitationally and are relatively close to one another. Stellar clusters are divided into two groups: **globular clusters**, containing large numbers of mainly old, evolved stars, and **open clusters**, containing smaller numbers of young stars (some are very hot) that are further apart, Figure D.1.

Very large numbers of stars and stellar clusters (about 200 billion of them) make up our **galaxy**, the Milky Way, a huge assembly of stars that are kept together by gravity. A galaxy with spiral arms (similar to the one in Figure D.2a), it is about 120 000 light years across; the arm in which our solar system is located can be seen on a clear dark night as the spectacular ‘milky’ glow of millions of stars stretching in a band across the sky.

As we leave our galaxy behind and enter intergalactic space, we find that our galaxy is part of a group of galaxies – a **cluster** (such as the one shown in Figure D.2b), known as the Local Group. There are about 30 galaxies in the Local Group, the nearest being the Large Magellanic

Learning objectives

- Describe the main objects comprising the universe.
- Describe the nature of stars.
- Understand astronomical distances.
- Work with the method of parallax.
- Define luminosity and apparent brightness and solve problems with these quantities and distance.

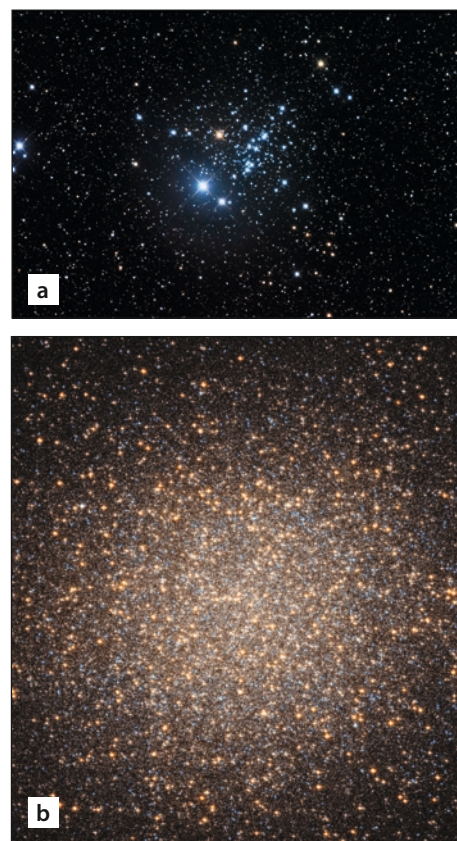


Figure D.1 a The open cluster M36; b the globular cluster M13.

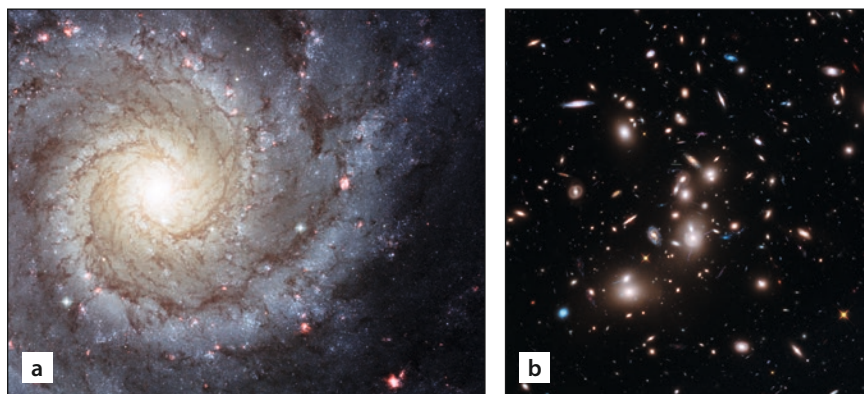


Figure D.2 a The spiral galaxy M74; b the galaxy cluster Abell 2744.

Cloud at a distance of about 160 000 light years. In this group, we also find the Andromeda galaxy, a spiral galaxy like our own and the largest member of the Local Group. Andromeda is expected to collide with the Milky Way in 4 billion years or so.

As we move even further out, we encounter collections of clusters of galaxies, known as **superclusters**. If we look at the universe on a really large scale, more than 10^8 light years, we then see an almost uniform distribution of matter. At such enormous scales, every part of the universe looks the same.

Binary star	Two stars orbiting a common centre
Black dwarf	The remnant of a white dwarf after it has cooled down. It has very low luminosity
Black hole	A singularity in space time; the end result of the evolution of a very massive star
Brown dwarf	Gas and dust that did not reach a high enough temperature to initiate fusion. These objects continue to compact and cool down
Cepheid variable	A star of variable luminosity. The luminosity increases sharply and falls off gently with a well-defined period. The period is related to the absolute luminosity of the star and so can be used to estimate the distance to the star
Cluster of galaxies	Galaxies close to one another and affecting one another gravitationally, behaving as one unit
Comet	A small body (mainly ice and dust) orbiting the Sun in an elliptical orbit
Constellation	A group of stars in a recognisable pattern that <i>appear</i> to be near each other in space
Dark matter	Generic name for matter in galaxies and clusters of galaxies that is too cold to radiate. Its existence is inferred from techniques other than direct visual observation
Galaxy	A collection of a very large number of stars mutually attracting one another through the gravitational force and staying together. The number of stars in a galaxy varies from a few million in dwarf galaxies to hundreds of billions in large galaxies. It is estimated that 100 billion galaxies exist in the observable universe
Interstellar medium	Gases (mainly hydrogen and helium) and dust grains (silicates, carbon and iron) filling the space between stars. The density of the interstellar medium is very low. There is about one atom of gas for every cubic centimetre of space. The density of dust is a trillion times smaller. The temperature of the gas is about 100 K
Main-sequence star	A normal star that is undergoing nuclear fusion of hydrogen into helium. Our Sun is a typical main-sequence star
Nebula	Clouds of 'dust', i.e. compounds of carbon, oxygen, silicon and metals, as well as molecular hydrogen, in the space in between stars
Neutron star	The end result of the explosion of a red supergiant; a very small star (a few tens of kilometres in diameter) and very dense. This is a star consisting almost entirely of neutrons. The neutrons form a superfluid around a core of immense pressure and density. A neutron star is an astonishing macroscopic example of microscopic quantum physics
Planetary nebula	The ejected envelope of a red giant star
Red dwarf	A very small star with low temperature, reddish in colour
Red giant	A main-sequence star evolves into a red giant – a very large, reddish star. There are nuclear reactions involving the fusion of helium into heavier elements
Stellar cluster	A group of stars that are physically near each other in space, created by the collapse of a single gas cloud
Supernova (Type Ia)	The explosion of a white dwarf that has accreted mass from a companion star exceeding its stability limit
Supernova (Type II)	The explosion of a red supergiant star: The amount of energy emitted in a supernova explosion can be staggering – comparable to the total energy radiated by our Sun in its entire lifetime!
White dwarf	The end result of the explosion of a red giant. A small, dense star (about the size of the Earth), in which no nuclear reactions take place. It is very hot but its small size gives it a very low luminosity

Table D.1 Definitions of terms.

Worked example

D.1 Take the density of interstellar space to be one atom of hydrogen per cm^3 of space. How much mass is there in a volume of interstellar space equal to the volume of the Earth? Give an order-of-magnitude estimate without using a calculator.

The volume of the Earth is

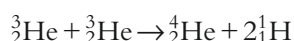
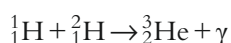
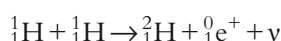
$$\begin{aligned} V &\approx \frac{4}{3}\pi R^3 \\ &\approx \frac{4}{3} \times 3 \times (6 \times 10^6)^3 \text{ m}^3 \\ &\approx 4 \times 200 \times 10^{18} \\ &\approx 10^{21} \text{ m}^3 \end{aligned}$$

The number of atoms in this volume is $10^{21} \times 10^6 = 10^{27}$ atoms of hydrogen (one atom in a cubic cm implies 10^6 atoms in a cubic metre). This corresponds to a mass of

$$10^{27} \times 10^{-27} \text{ kg} \approx 1 \text{ kg}.$$

D1.2 The nature of stars

A star such as our own Sun radiates an enormous amount of power into space – about 10^{26} J s^{-1} . The source of this energy is nuclear fusion in the interior of the star, in which nuclei of hydrogen fuse to produce helium and energy. Because of the **high temperatures** in the interior of the star, the electrostatic repulsion between protons can be overcome, allowing hydrogen nuclei to come close enough to each other to fuse. Because of the **high pressure** in stellar interiors, the nuclei are sufficiently close to each other to give a high probability of collision and hence fusion. The sequence of nuclear fusion reactions that take place is called the **proton–proton cycle** (Figure D.3):



The net result of these reactions is that four hydrogen nuclei turn into one helium nucleus (to see this multiply the first two reactions by 2 and add side by side). Energy is released at each stage of the cycle, but most of it is released in the third and final stage. The energy produced is carried away by the photons (and neutrinos) produced in the reactions. As the photons move outwards they collide with the surrounding material, creating a **radiation pressure** that opposes the gravitational pressure arising from the mass of the star. In the outer layers, convection currents also carry the energy outwards. In this way, the balance between radiation and gravitational pressures keeps the star in equilibrium (Figure D.4).

Nuclear fusion provides the energy that is needed to keep the star hot, so that the radiation pressure is high enough to oppose further gravitational contraction, and at the same time to provide the energy that the star is radiating into space.

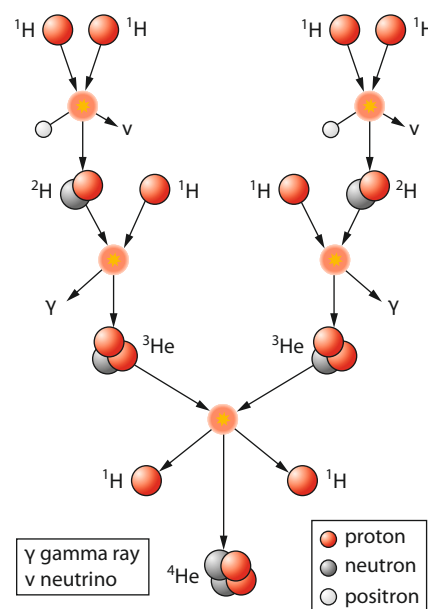


Figure D.3 The proton–proton cycle of fusion reactions.

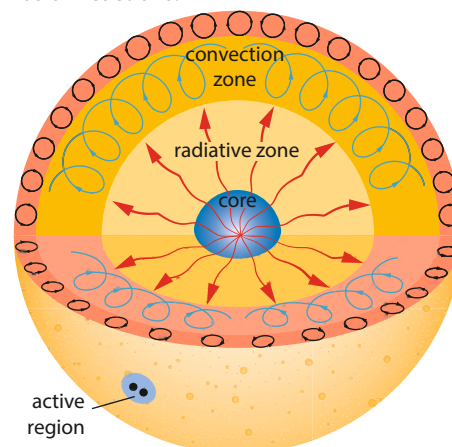


Figure D.4 The stability of a star depends on equilibrium between two opposing forces: gravitation, which tends to collapse the star, and radiation pressure, which tends to make it expand.

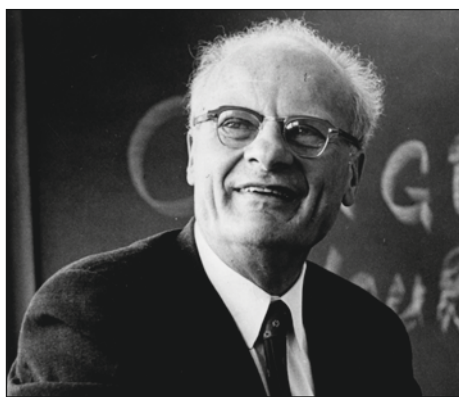


Figure D.5 Hans Bethe, who unravelled the secrets of energy production in stars.

Exam tip

$$1 \text{ AU} = 1.5 \times 10^{11} \text{ m}$$

$$1 \text{ ly} = 9.46 \times 10^{15} \text{ m}$$

$$1 \text{ pc} = 3.09 \times 10^{16} \text{ m} = 3.26 \text{ ly}$$

Many of the details of nuclear fusion reactions in stars were worked out by the legendary Cornell physicist Hans Bethe (1906–2005) (Figure D.5).

D1.3 Astronomical distances

In astrophysics, it is useful to have a more convenient unit of distance than the metre!

A **light year** (ly) is the distance travelled by light in one year. Thus:

$$\begin{aligned} 1 \text{ ly} &= 3 \times 10^8 \times 365 \times 24 \times 60 \times 60 \text{ m} \\ &= 9.46 \times 10^{15} \text{ m} \end{aligned}$$

Also convenient for measuring large distances is the **parsec** (pc), a unit that will be properly defined in Section D1.4:

$$1 \text{ pc} = 3.26 \text{ ly} = 3.09 \times 10^{16} \text{ m}$$

Yet another convenient unit is the **astronomical unit** (AU), which is the average radius of the Earth's orbit around the Sun:

$$1 \text{ AU} = 1.5 \times 10^{11} \text{ m}$$

The average distance between stars in a galaxy is about 1 pc. The distance to the nearest star (Proxima Centauri) is approximately $4.2 \text{ ly} = 1.3 \text{ pc}$. A simple message sent to a civilisation on Proxima Centauri would thus take 4.2 yr to reach it and an answer would arrive on Earth another 4.2 yr later.

The average distance between galaxies varies from about 100 kiloparsecs (kpc) for galaxies within the same cluster to a few megaparsecs (Mpc) for galaxies belonging to different clusters.

Worked examples

D.2 The Local Group is a cluster of some 30 galaxies, including our own Milky Way and the Andromeda galaxy. It extends over a distance of about 1 Mpc. Estimate the average distance between the galaxies of the Local Group.

Assume that a volume of

$$\begin{aligned} V &\approx \frac{4}{3}\pi R^3 \\ &\approx \frac{4}{3} \times 3 \times (0.5)^3 \text{ Mpc}^3 \\ &\approx 0.5 \text{ Mpc}^3 \end{aligned}$$

is uniformly shared by the 30 galaxies. Then to each there corresponds a volume of

$$\frac{0.5}{30} \text{ Mpc}^3 = 0.017 \text{ Mpc}^3$$

The linear size of each volume is thus

$$\begin{aligned} \sqrt[3]{0.017 \text{ Mpc}^3} &\approx 0.3 \text{ Mpc} \\ &= 300 \text{ kpc} \end{aligned}$$

so we may take the average separation of the galaxies to be about 300 kpc.

D.3 The Milky Way galaxy has about 2×10^{11} stars. Assuming an average stellar mass equal to that of the Sun, estimate the mass of the Milky Way.

The mass of the Sun is 2×10^{30} kg, so the Milky Way galaxy has a mass of about
 $2 \times 10^{11} \times 2 \times 10^{30} \text{ kg} = 4 \times 10^{41} \text{ kg}$

D.4 The observable universe contains some 100 billion galaxies. Assuming an average galactic mass comparable to that of the Milky Way, estimate the mass of the observable universe.

The mass is $100 \times 10^9 \times 4 \times 10^{41} \text{ kg} = 4 \times 10^{52} \text{ kg}$.

D1.4 Stellar parallax and its limitations

The **parallax** method is a means of measuring astronomical distances. It takes advantage of the fact that, when an object is viewed from two different positions, it appears displaced relative to a fixed background. If we measure the angular position of a star and then repeat the measurement some time later, the two positions will be different relative to a background of very distant stars, because in the intervening time the Earth has moved in its orbit around the Sun. We make two measurements of the angular position of the star six months apart; see Figure D.6.

The distance between the two positions of the Earth is $D = 2R$, the diameter of the Earth's orbit around the Sun ($R = 1.5 \times 10^{11} \text{ m}$). The distance d to the star is given by

$$\tan p = \frac{R}{d} \Rightarrow d = \frac{R}{\tan p}$$

Since the parallax angle is very small, $\tan p \approx p$, where the parallax p is measured in radians, and so $d = \frac{R}{p}$.

The parallax angle (shown in Figure D.6) is the angle, at the position of the star, that is subtended by a distance equal to the radius of the Earth's orbit around the Sun (1 AU).

The parallax method can be used to define a common unit of distance in astronomy, the **parsec**. One parsec (from **parallax second**) is the distance to a star whose parallax is 1 arc second, as shown in Figure D.7. An arc second is $1/3600$ of a degree.

In conventional units,

$$1 \text{ pc} = \frac{1 \text{ AU}}{1''} = \frac{1.5 \times 10^{11}}{\left(\frac{2\pi}{360}\right)\left(\frac{1}{3600}\right)} \text{ m} = 3.09 \times 10^{16} \text{ m}$$

(The factor of $\frac{2\pi}{360}$ converts degrees to radians.)

If the parallax of a star is known to be p arc seconds, its distance is

$$d \text{ (in parsecs)} = \frac{1}{p} \text{ (in arc seconds)}.$$

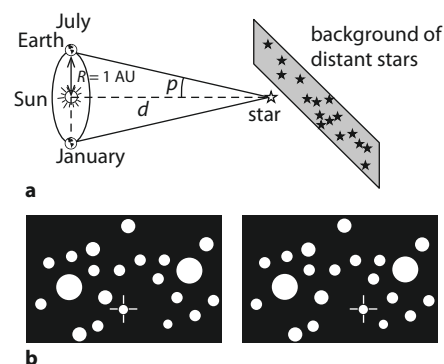


Figure D.6 **a** The parallax of a star. **b** Two 'photographs' of the same region of the sky taken six months apart. The position of the star (indicated by a cross) has shifted, relative to the background stars, in the intervening six months.

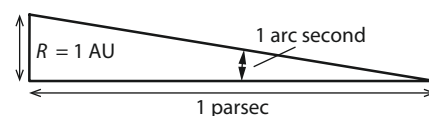


Figure D.7 Definition of a parsec: 1 parsec is the distance at which 1 AU subtends an angle of 1 arc second.

Exam tip

You will not be asked to provide these derivations in an exam. You should just know that d (in parsecs) $= \frac{1}{p}$ (in arc seconds).

You must also understand the limitations of this method.



Ancient methods are still useful!

Astrophysicists still use an ancient method of measuring the apparent brightness of stars. This is the **magnitude scale**. Given a star of apparent brightness b , we assign to that star an **apparent magnitude** m , defined by

$$\frac{b}{b_0} = 100^{-m/5}$$

The value $b_0 = 2.52 \times 10^{-8} \text{ W m}^{-2}$ is taken as the reference value for apparent brightness. Taking logarithms (to base 10) gives the equivalent form

$$m = -\frac{5}{2} \log\left(\frac{b}{b_0}\right)$$

Since $100^{1/5} = 2.512$, the first equation above can also be written as

$$\frac{b}{b_0} = 2.512^{-m}$$

If the star is too far away, however, the parallax angle is too small to be measured and this method fails. Typically, measurements from observatories on Earth allow distances up to 100 pc to be determined by the parallax method, which is therefore mainly used for nearby stars. Using measurements from satellites without the distortions caused by the Earth's atmosphere (turbulence, and variations in temperature and refractive index), much larger distances can be determined using the parallax method. The Hipparcos satellite (launched by ESA, the European Space Agency, in 1989) measured distances to stars 1000 pc away; Gaia, launched by ESA in December 2013, is expected to do even better, extending the parallax method to distances beyond 100 000 pc!

Table D.2 shows the five nearest stars (excluding the Sun).

Star	Distance/ly
Proxima Centauri	4.3
Barnard's Star	5.9
Wolf 359	7.7
Lalande 21185	8.2
Sirius	8.6

Table D.2 Distances to the five nearest stars.

D1.5 Luminosity and apparent brightness

Stars are assumed to radiate like black bodies. For a star of surface area A and absolute surface temperature T , we saw in Topic 8 that the power radiated is

$$L = \sigma AT^4$$

where the constant σ is the Stefan–Boltzmann constant ($\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$).

The power radiated by a star is known in astrophysics as the **luminosity**. It is measured in watts.



Consider a star of luminosity L . Imagine a sphere of radius d centred at the location of the star. The star radiates uniformly in all directions, so the energy radiated in 1 s can be thought of as distributed over the surface of this imaginary sphere. A detector of area a placed somewhere on this sphere will receive a small fraction of this total energy (see Figure D.8a).

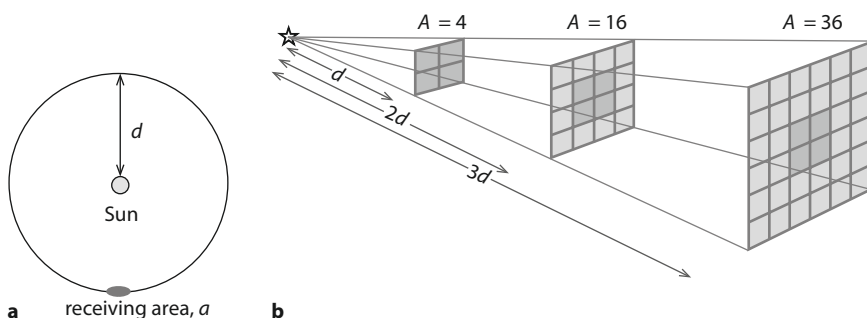


Figure D.8 **a** The Sun's energy is distributed over an imaginary sphere of radius equal to the distance between the Sun and the observer. The observer thus receives only a very small fraction of the total energy, equal to the ratio of the receiver's area to the total area of the imaginary sphere. **b** The inverse square law.

This fraction is equal to the ratio of the detector area a to the total surface area of the sphere; that is, the received power is $\frac{aL}{4\pi d^2}$.

This shows that the apparent brightness is directly proportional to the luminosity, and varies as the inverse square of the star's distance (see Figure D.8b). By combining the formula for luminosity with that for apparent brightness, we see that

$$b = \frac{\sigma AT^4}{4\pi d^2}$$

Apparent brightness is easily measured (with a charge-coupled device, or CCD). If we also know the distance to a star, then we can determine its luminosity. Knowing the luminosity of a star is important because it tells a lot about the nature of the star.

Exam tip

In many problems you will need to know that the surface area of a sphere of radius R is $A = 4\pi R^2$.

The received power per unit area is called the **apparent brightness** and is given by

$$b = \frac{L}{4\pi d^2}$$

The unit of apparent brightness is W m^{-2} .

Exam tip

Apparent brightness in astrophysics is what is normally called intensity in physics.

Worked examples

D.5 The radius of star A is three times that of star B and its temperature is double that of B. Find the ratio of the luminosity of A to that of B.

$$\begin{aligned} \frac{L_A}{L_B} &= \frac{\sigma 4\pi (R_A)^2 T_A^4}{\sigma 4\pi (R_B)^2 T_B^4} \\ &= \frac{(R_A)^2 T_A^4}{(R_B)^2 T_B^4} \\ &= \frac{(3R_B)^2 (2T_B)^4}{(R_B)^2 T_B^4} \\ &= 3^2 \times 2^4 = 144 \end{aligned}$$

D.6 The stars in Worked example **D.5** have the same apparent brightness when viewed from the Earth. Calculate the ratio of their distances.

$$\begin{aligned}\frac{b_A}{b_B} &= 1 \\ &= \frac{L_A/(4\pi d_A^2)}{L_B/(4\pi d_B^2)} \\ &= \frac{L_A}{L_B} \frac{d_B^2}{d_A^2} \\ \Rightarrow \frac{d_A}{d_B} &= 12\end{aligned}$$

D.7 The apparent brightness of a star is $6.4 \times 10^{-8} \text{ W m}^{-2}$. Its distance is 15 ly. Find its luminosity.

We use $b = \frac{L}{4\pi d^2}$ to find

$$\begin{aligned}L &= b4\pi d^2 = \left(6.4 \times 10^{-8} \frac{\text{W}}{\text{m}^2}\right) \times 4\pi \times (15 \times 9.46 \times 10^{15})^2 \text{ m}^2 \\ &= 1.6 \times 10^{28} \text{ W}\end{aligned}$$

D.8 A star has half the Sun's surface temperature and 400 times its luminosity. Estimate the ratio of the star's radius to that of the Sun. The symbol R_\odot stands for the radius of the Sun.

We have that

$$\begin{aligned}400 &= \frac{L}{L_\odot} = \frac{\sigma 4\pi (R^2) T^4}{\sigma 4\pi (R_\odot)^2 T_\odot^4} = \frac{(R^2)(T/2)^4}{(R_\odot)^2 T_\odot^4} = \frac{R^2}{(R_\odot)^2 16} \\ \Rightarrow \frac{R^2}{(R_\odot)^2} &= 16 \times 400 \\ \Rightarrow \frac{R}{R_\odot} &= 80\end{aligned}$$

Nature of science

Reasoning about the universe

Over millennia, humans have mapped the planets and stars, recording their movements and relative brightness. By applying the simple idea of parallax, the change in position of a star in the sky at times six months apart, astronomers could begin to measure the distances to stars. Systematic measurement of the distances and the relative brightness of stars and galaxies, with increasingly sophisticated tools, has led to an understanding of a universe that is so large it is difficult to imagine.



? Test yourself

- 1 Determine the distance to Procyon, which has a parallax of $0.285''$.
- 2 The distance to Epsilon Eridani is 10.8 ly. Calculate its parallax angle.
- 3 Betelgeuse has an angular diameter of $0.016''$ (that is, the angle subtended by the star's diameter at the eye of an observer) and a parallax of $0.0067''$.
 - a Determine the distance of Betelgeuse from the Earth.
 - b What is its radius in terms of the Sun's radius?
- 4 A neutron star has an average density of about $10^{17} \text{ kg m}^{-3}$. Show that this is comparable to the density of an atomic nucleus.
- 5 A sunspot near the centre of the Sun is found to subtend an angle of 4.0 arc seconds. Find the diameter of the sunspot.
- 6 The resolution of the Hubble Space Telescope is about 0.05 arc seconds. Estimate the diameter of the smallest object on the Moon that can be resolved by the telescope. The Earth-moon distance is $D = 3.8 \times 10^8 \text{ m}$.
- 7 The Sun is at a distance of 28 000 ly from the centre of the Milky Way and revolves around the galactic centre with a period of about 211 million years. Estimate from this information the orbital speed of the Sun and the total mass of the Milky Way.
- 8
 - a Describe, with the aid of a clear diagram, what is meant by the **parallax method** in astronomy.
 - b Explain why the parallax method fails for stars that are very far away.
- 9 The light from a star a distance of 70 ly away is received on Earth with an apparent brightness of $3.0 \times 10^{-8} \text{ W m}^{-2}$. Calculate the luminosity of the star.
- 10 The luminosity of a star is $4.5 \times 10^{28} \text{ W}$ and its distance from the Earth is 88 ly. Calculate the apparent brightness of the star.
- 11 The apparent brightness of a star is $8.4 \times 10^{-10} \text{ W m}^{-2}$ and its luminosity is $6.2 \times 10^{32} \text{ W}$. Calculate the distance to the star in light years.
- 12 Two stars have the same size but one has a temperature that is four times larger.
 - a Estimate how much more energy per second the hotter star radiates.
 - b The apparent brightness of the two stars is the same; determine the ratio of the distance of the cooler star to that of the hotter star.
- 13 Two stars are the same distance from the Earth and their apparent brightnesses are $9.0 \times 10^{-12} \text{ W m}^{-2}$ (star A) and $3.0 \times 10^{-13} \text{ W m}^{-2}$ (star B). Calculate the ratio of the luminosity of star A to that of star B.
- 14 Take the surface temperature of our Sun to be 6000 K and its luminosity to be $3.9 \times 10^{26} \text{ W}$. Find, in terms of the solar radius, the radius of a star with:
 - a temperature 4000 K and luminosity $5.2 \times 10^{28} \text{ W}$
 - b temperature 9250 K and luminosity $4.7 \times 10^{27} \text{ W}$.
- 15 Two stars have the same luminosity. Star A has a surface temperature of 5000 K and star B a surface temperature of 10 000 K.
 - a Suggest which is the larger star and by how much.
 - b The apparent brightness of A is double that of B; calculate the ratio of the distance of A to that of B.
- 16 Star A has apparent brightness $8.0 \times 10^{-13} \text{ W m}^{-2}$ and its distance is 120 ly. Star B has apparent brightness $2.0 \times 10^{-15} \text{ W m}^{-2}$ and its distance is 150 ly. The two stars have the same size. Calculate the ratio of the temperature of star A to that of star B.
- 17 Calculate the apparent brightness of a star of luminosity $2.45 \times 10^{28} \text{ W}$ and a parallax of $0.034''$.