Investigating the Pressure Under Water as a Function of Depth

Take the Plunge

I am an avid swimmer, and one of my favorite challenges is to swim the entire length of the school's pool underwater. To help facilitate this I force myself to go as deep as possible, so that when I return to the surface I also continue moving along the horizontal, reaching the far end of the pool still submerged.

When performing my Herculean feat I noticed that as I go deeper under water the pressure on my ears increases. I am feeling the 'weight' of the water above me. In fact, the weight or force over a give surface area is **pressure**, so as the depth of water increases to too does the pressure increase. I wanted to research this experience of mine from the view of physics. My research project then is to confirm, quantitatively, the relationship of water depth and pressure.

Relevance

My research is interesting not only from my personal experience but from other perspectives too. Ask yourself: Why do bubbles at the bottom of the pool increase in size as they float upwards? The answer is that deep in the pool the pressure is greater and compresses the air into a small sphere; then, while approaching the surface, the water pressure is less and the bubble expands approaching one atmospheric pressure when it pops at the surface. Another example from the real world concerns the more serious problem of decompression sickness that occurs when a skin diver returns to the surface too fast. The 'bends' describe the condition arising from dissolved gases coming out of solution into bubbles inside the body on depressurization.[1] And in a less serious note, I have noticed that sometimes my ears 'pop' when I dive into deep water.[2]

Research

We were told that going underwater to a depth of about 33 feet would double the pressure.[3] This translates as about one atmospheric pressure per ten meters of depth. It would seem reasonable to say there is a linear relationship between depth and pressure, and a few textbooks confirm this.[4] An expression for the pressure p at a depth h in a liquid of density ρ where gravity is g, is given by $p = h\rho g$.

When the depth h is measured in metres (m), the density ρ is in kilograms per cubic meter $\left(\mathrm{kg\,m^{-3}}\right)$ and gravity g is in newtons per kilograms $\left(\mathrm{N\,kg^{-1}}\right)$, the pressure p is in units of pascals (Pa). I will us kilopascals, kPA, or 10^3 Pa.

At the surface of the water, the pressure due to the water is of course zero, but the atmospheric pressure is still there, and we can denoted this as *P*. This means that when submerged underwater the total pressure on me is due to the atmosphere and the water, so the relevant equation becomes:

$$p_{(\mathrm{h})} = p_{\mathrm{water}} + P_{\mathrm{atmosphere}} = h\rho g + P_{\mathrm{atm}}$$

where $p_{(\mathrm{h})}$ is the total or absolute pressure at a depth h, and this is just the sum of the pressure due to the water plus the atmospheric pressure at the surface of the water. I will devise a simple experiment to confirm this for my physics investigation.

Technical Terms

Atmospheric Pressure is the pressure due to the earth's atmosphere and is often denoted ATM, or 1 atmospheric pressure. At normal temperatures like 15°C and at sea level this is about 101.325 kPa, where one Pascal is one Newton of force applied to one square meter of area.[5]

Ambient Pressure is the pressure due to water at a given depth. This is the pressure normally displayed on a scuba divers depth gauge. This is the same as "gauge pressure".

Gauge Pressure refers to the pressure based on the depth of water alone. At the surface of water, gauge pressure is zero. One uses gauge pressure when measuring air pressure in a car tire, for example.

A**bsolute Pressure** is the combined pressure due to water at a given depth plus the atmosphere. This is what I will measure in my investigation.

Atmospheric Pressure

Although normal atmospheric pressure is about 100 kPa, my school is located at nearly 2000 meters up in the Rocky Mountains of the United States. The 'blanket' of air covering us here is less than that measured at sea level. In fact, when I buy a bag of crisps (potato chips) in New York and bring them back to campus, the bag has expanded noticeably. Moreover, when I first open my shampoo (closed at near sea level) soap squirts out due to the higher pressure inside the container compared to the lower air pressure here. Therefore I do not expect to have standard air pressure in my experiment. I found the following graph (figure 1) of air pressure and altitude on the Internet.

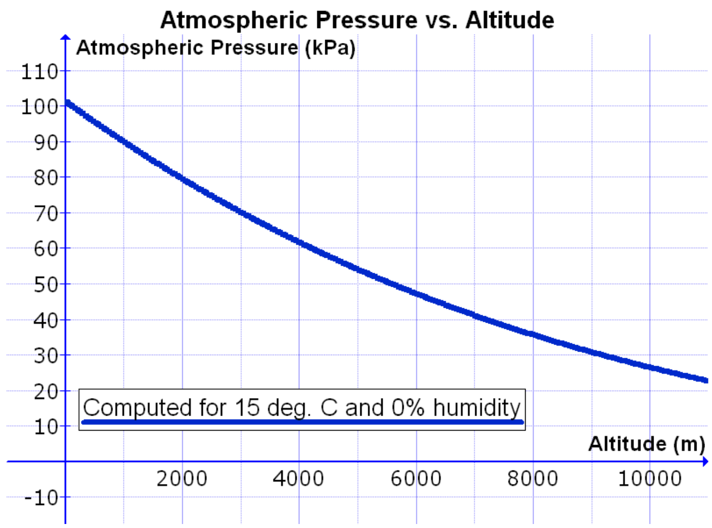


Figure 1: Atmospheric Pressure as Function of Altitude[6]

According to this graph, at my altitude the normal pressure should be about 80 kPa; a basic measurement reveals this is correct.

Atmospheric Pressure Measured

The instructions given my Vernier's Barometer probe tell us to ether measure the atmospheric variations over time, to study changes in the weather, or for water depth, to connect a rubber tube to the sensor input and immerse the end of the rubber tube in water (see page 6 of the specification sheet, web site in footnote)[7]. This struck me as somewhat inappropriate as it ignores the effect of the air being compressed under the increased water pressure. This is simply Boyle's law. My method takes account of this.

First I set up my computer and the *LabPro* interface using *LoggerPro* software. I recorded the atmospheric pressure. It noticed slight variations over time, either random errors or changes in pressure. Because of this I recorded the atmospheric pressure every second for five minutes and then had the computer determine (using the statistical analysis function) the average value and the standard deviation of the average value.

My results gave a mean value of **82.6954686 kPa** with a **SD of 0.006994**. Rounded off to significant figures, this is an atmospheric pressure of **(82.695 ± 0.007) kPa**. This is an uncertainty of about 8 x 10^{-3} % which is insignificant. My classroom pressure of about 83 kPa is in agreement with the Wikipedia graph in figure 1 above.

Vernier's Barometer Sensor, Model BAR-BTA

According to the manufacture, the sensor's resolution is 1.0026 x 10^{-4} atm. **This is an uncertainty value of ±0.1%.** Such a value proves to be too small to include on my graph, so I will only graph uncertainty bars for the depth measurements.

Figure 2: Vernier's Barometer Pressure Sensor (which measures absolute pressure)[8]

Experimental Method

In order to measure the pressure at a given water depth, I connected the rubber hose to a glass tube. To measure the water depth I submerge the glass tube along the vertical and carefully observe the water level inside the tube. The depth is the distance from the water surface to the water-air interface inside the tube. The deeper the glass tube, the more the air inside the tube increases in pressure. According to Boyle's law, the volume of air decreases, and so it would be inappropriate to assume that the end of the glass tube is the water depth. This difference is crucial for accurate depth measurements.

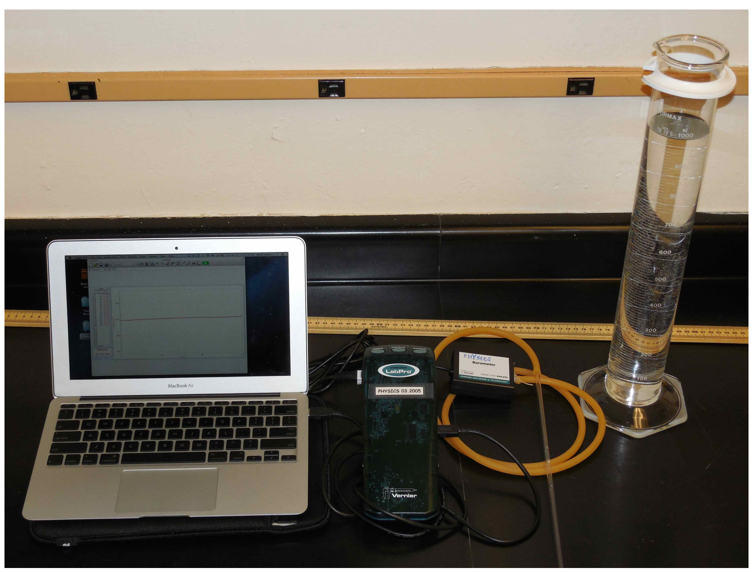


Figure 3: Experimental Setup
Computer, Vernier's LabPro, Vernier's Barometer Sensor, Rubber Tube, Glass Tube, 1000 mL Graduated Cylinder filled with water, Meter Rule

I used the 100 mL marks on the graduated measuring cylinder to align the air-water boundary in the glass tube, thus determining the depth (for the pressure measurement) when measured from the water level surface. I then used a meter rule to measure the depth. Note how much longer the glass tube is compared to the water-air level.

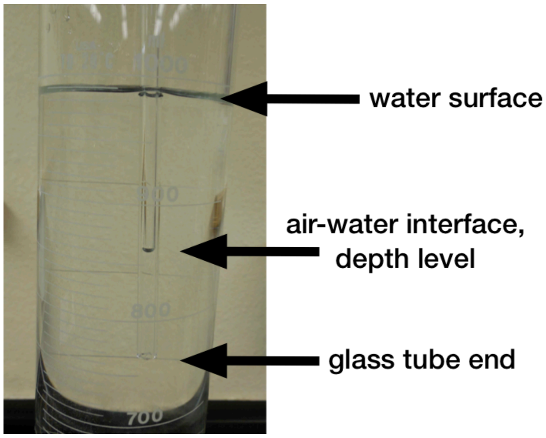


Figure 4: Close Up Photo of Glass Tube Immersed in Water

Note how the end of the glass tube is much lower in the water than the water-air interface within the glass tube. This is due to the water pressure compressing the air and, according to Boyle's law, reducing the air volume. The depth is measured from the water surface to this water-air interface level.

Data and Analysis

	Data Set	
	Depth (m)	Absolute Pressure (kPa)
1	0.000	82.69547
2	0.032	82.99423
3	0.065	83.29497
4	0.101	83.69550
5	0.134	83.99604
6	0.167	84.39555
7	0.203	84.69622
8	0.240	85.09511
9	0.275	85.39477
10	0.313	85.69547

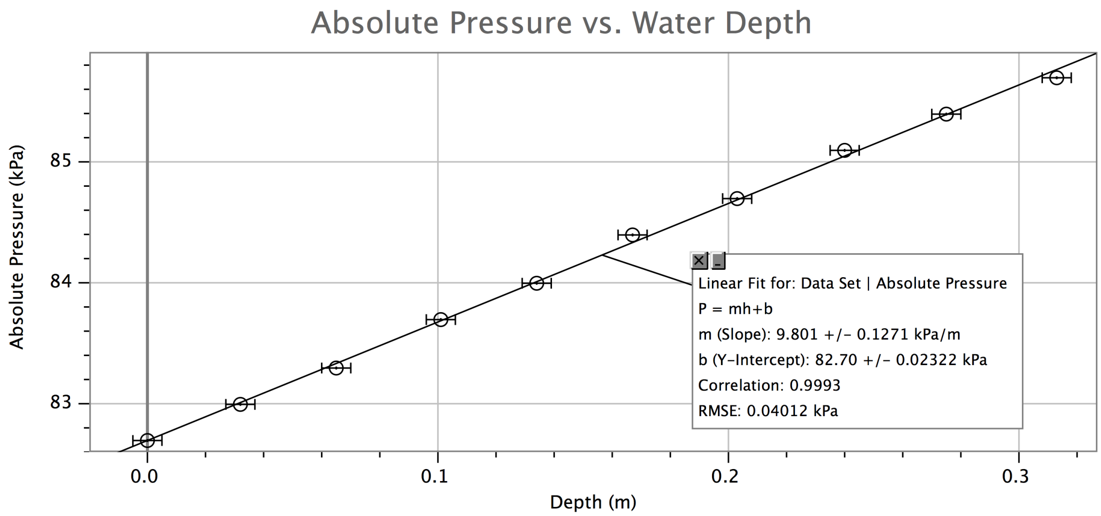
Figure 5: Raw Data

The uncertainty in the depth was estimated to be ±5mm. The least count of the meter rules is 1 mm but aligning the air-water interface (where there is a slight miscues curve) and keeping the ruler vertical, can all add up to an estimated uncertainty of one-half a centimeter (at the worst). It is probably not this bad but I will use this value but my experimental uncertainty comes from the graph's best-fit line.

The odd water depth measurement come from the initial alignment of the water-air interface within the tube at the various 100 mL graduated lines.

Figure 6: Experimental Graph

The best-fit linear line and standard deviation uncertainty were determined by the LoggerPro graphing program. The uncertainty bars are for depth measurements only, as the pressure uncertainty is too small to consider.



The altitude here is nearly 2 km, and the local university has measured gravity to be 9.80 m s^{-2}. Using this and the textbook theory of water pressure with the standard density of water, we see that in theory the gradient of a pressure against depth graph should have a value of 9.80 kPa m^{-1}.

$$p = h\rho g \rightarrow \frac{p}{h} = \rho g$$

$$\text{theory gradient of graph}\left(\frac{\Delta p}{\Delta h}\right) = \rho g = \left(1000\,\text{kg m}^{-3}\right)\left(9.80\,\text{m s}^{-2}\right) = 9800\,\text{Pa m}^{-1}$$

$$\text{gradient theory} = 9.80\,\text{kPa m}^{-1}$$

The **experimental gradient** of my graph is = 9.80 kPa m^{-1}.

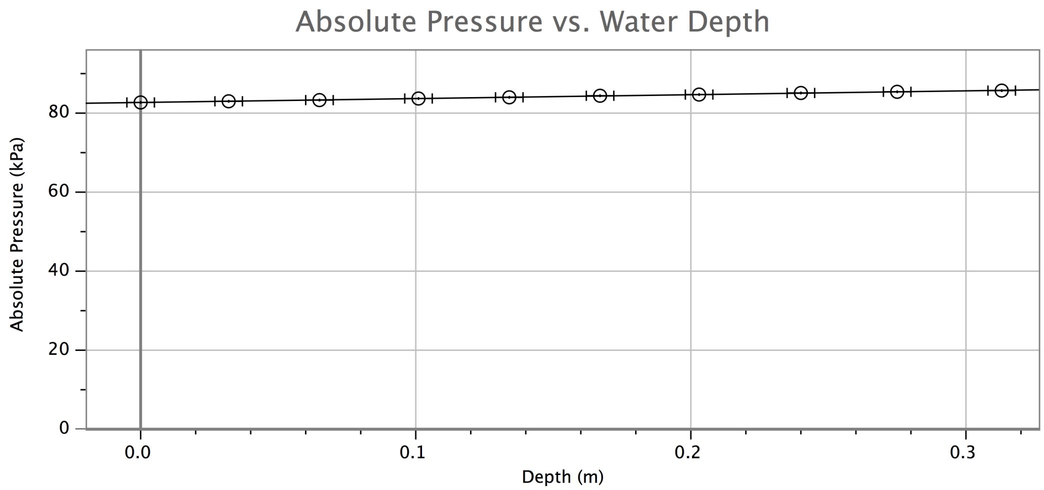
Although this result seems unlikely (perhaps too good to be true), when you consider the experimental uncertainties the ‘perfect’ match is not perfect. *LogerPro* software analysis drew the best straight-line for the data with a gradient of 9.801 ± 0.127 or about 9.8 ± 0.1, which represents about 1% error.

Note also that the line touches all data point uncertainty bars, and the correlation is 0.9993—no finer value could be expected.

The *y*-intercept represents the atmospheric pressure in the room at the time of the experiment. The software extrapolated this to be (82.70 ± 0.02) kPa. The directly measured room air presume of 82.695 kPa is well within the range here of 82.68 to 82.72 kPa.

Here is the same graph data but using a vertical scale from zero. You can see the slight but nonetheless real increase of pressure due to water depth.

Figure 7: Graph



Repeated Measurements

I repeated the entire experiment on another day. Because the initial room air pressure was a little different and because it was hard to reproduce the same depth measurement (to under a mm) I decided not to combine the two-data set as I usually do in class experiments with repeated measures, but instead I constructed a second graph and determined the gradient. In my second trial I obtained a gradient of 9.78327 ± 0.120843, or rounded to significant figures, (9.78 ±0.12) kPa m^{-1}. This result also confirms the relationship of depth and pressure, but in my second trial the standard deviation uncertainty is about 1.4%, a little more than my first trial of just 1%. Both results are well within experimental uncertainties. There is no need for a third trail. Both trials confirm the theory nicely.

Conclusion.

Within the limited range of the depth of the measuring cylinder, a depth from zero to only about 30 cm, I found a consistent and linear increase in absolute pressure. My results were good to just a little over 1% uncertainty, and the trend line has a very high correlation. My range of pressure difference was only 3 kPa, a range of only 3% but this was enough to discern the trend and confirm the relevant equation.

The following (figure 8) illustrates the pressure and depth principle for a much greater depth than I was able to measure.

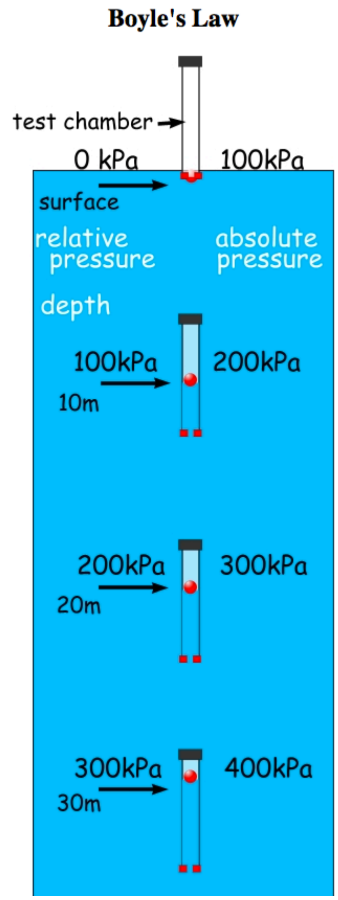


Figure 8. Pressure and Water Depth[9]

Improvements and Extensions

(1) I always measured from the water surface. As the glass rod was inserted, it displaced some water thus raising the water surface. I believe that my measurement took account of this, but it is a point to be noted. I used my eye to keep the ruler vertical, and held it against the cylinder, a safe assumption. However, if I could measure the water depth to a higher degree of precision I would reduce my 1% uncertainty. Perhaps a vernier caliper fixed to the cylinder side could be adjusted, aligned at both ends, and a depth read to a fraction of a millimeter.[10]

(2) For a relevant extension of this study, I would find a deeper water source, perhaps the school swimming pool. This would introduce more problems of depth measurements but it would extend the range of pressure. Would love to have a water depth of 10 meters to find twice the total pressure.

(3) Finally, I could extend my study to different liquids, such as different densities of different oils.

Footnote References

#1 http://en.wikipedia.org/wiki/Decompression_sickness

#2 http://www.livestrong.com/article/402126-ear-pressure-from-swimming/

#3 http://van.physics.illinois.edu/qa/listing.php?id=2233

#4 See page 206 in “Higher Physics” by Jim Jardine (Heinemann Educational Books, 1983), and see pages 217 to 220 in “Physics: A Textbook for Advanced Level Students” 2nd edition by Tom Duncan, John Murray Publishers, 1987.

#5 http://en.wikipedia.org/wiki/Atmospheric_pressure#Standard_atmospheric_pressure

#6 http://en.wikipedia.org/wiki/File:Atmospheric_Pressure_vs._Altitude.png

#7 http://www.vernier.com/products/sensors/bar-bta/

#8 http://www.vernier.com/products/sensors/bar-bta/

#9 http://resources.yesican-science.ca/Bolyes_law/trans_results1.html

#10 http://en.wikipedia.org/wiki/Caliper

Please refer to the IA Criteria and descriptors as found in the Physics Course Guide, pages 143–147

The following marking grid is a guide to the criteria and descriptors.

Circle the best-fit indicator level for each descriptor.

Personal Engagement @ total ___ / 2

This criterion assesses the extent to which the student engages with the exploration and makes it their own. Personal engagement may be recognized in different attributes and skills. These could include addressing personal interests or showing evidence of independent thinking, creativity or initiative in the designing, implementation or presentation of the investigation.

Descriptor	0	1	2
evidence of personal engagement with exploration	standard not reached	limited with little independent thinking, initiative or insight	clear with significant independent thinking, initiative or creativity
justification given for choosing the research question and/or the topic under investigation	standard not reached	does not demonstrate personal significance, interest or curiosity	demonstrates personal significance, interest or curiosity
evidence of personal input and initiative in the designing, implementation or presentation of the investigation	standard not reached	little evidence	evidence

Exploration @ total ___ / 6

This criterion assesses the extent to which the student establishes the scientific context for the work, states a clear and focused research question and uses concepts and techniques appropriate to the Diploma Programme level. Where appropriate, this criterion also assesses awareness of safety, environmental, and ethical considerations.

Descriptor	0	1	2	3	4	5	6
topic of the investigation is identified and relevant research question described	standard not reached	some relevance is stated but it is not focused		relevant but not fully focused		relevant, fully focused and clearly described	
background information provided for the investigation	standard not reached	superficial or of limited relevance and does not aid the under-standing of the context of the investigation		mainly appropriate and relevant and aids the under-standing of the context of the investigation		entirely appropriate and relevant and enhances the understanding of the context of the investigation	
appropriate of methodology of the investigation, consideration of factors for reliability and sufficiency of data	standard not reached	limited to research question, few if any factors considered		mainly appropriate but some limits on significant factors		highly appropriate, all or nearly all factors are considered	
evidence of awareness of the significant safety, ethical or environmental issues that are relevant to the methodology of the investigation	standard not reached	limited awareness		some awareness		full awareness	

Analysis @ total ___ / 6

This criterion assesses the extent to which the student's report provides evidence that the student has selected, recorded, processed and interpreted the data in ways that are relevant to the research question and can support a conclusion.

Descriptor	0	1	2	3	4	5	6
raw data	standard not reached	insufficient to support a valid conclusion		relevant but incomplete. Could support a simple or partially valid conclusion		sufficient; could support a detailed and valid conclusion	
data processing, accuracy and consistent with data	standard not reached	some basic but too inaccurate or insufficient to lead to a valid conclusion		appropriate and sufficient raw data carried out that could lead to a conclusion but with significant inaccuracies and inconsistencies in processing		appropriate and sufficient data processing with accuracy so as to enable a conclusion to the research question to be drawn that is fully consistent with the experimental data	
impact of uncertainties on the analysis	standard not reached	little evidence of the impact of uncertainties		some evidence of the impact of uncertainties		full and appropriate evidence of the impact of uncertainties	
interpretation of processed data	standard not reached	incorrect or insufficient interpretation that may lead to an invalid or very incomplete conclusion		broadly valid interpretation leading to an incomplete or limited conclusion		correct interpretation allowing a completely valid and detailed conclusion	

Evaluation @ total ___ / 6

This criterion assess the extend to which the student's report provides evidence of evaluation of the investigation and the results with regard to the research question and the accepted scientific context.

Descriptor	0	1	2	3	4	5	6
conclusion statement	standard not reached	outlined but not relevant to the research question or not supported by the data presented		described, relevant to the research question and supported by the data presented		described in detail and justified, entirely relevant to the research question and fully supported by the data presented	
conclusion and accepted theory	standard not reached	superficially compared to the accepted scientific context		some relevant comparison to accepted scientific context		correctly described and justified through relevant comparison to the accepted scientific context	
strengths and weaknesses of the investigation, such as limitations of the data and sources of error, are discussed and provide evidence to a clear understanding of the methodological issues involved in establishing the conclusion	standard not reached	outlined but are restricted to an account of the practical or procedural issues faced		described and provide evidence of some awareness of the methodological issues involved in establishing the conclusion		discussed and provide evidence of a clear understanding of the method-ological issues involved in establishing the conclusion	
realistic and relevant suggestions for the improvement and extension of the investigation	standard not reached	very few outlined		some described		are discussed	

Communication @ total ___ / 4

This criterion assesses whether the investigation is presented and reported in a way that supports effective communication of the focus, process and outcomes.

Descriptor	0	1	2	3	4
presentation of the investigation	standard not reached	unclear, making it difficult to understand the focus, process and outcomes		clear, any errors do not hamper understanding of the focus, process and outcomes	
report structure	standard not reached	not well structured and is unclear: the necessary information on focus, process and outcomes is missing or is presented in an incoherent or disorganized way		well structured and clear: the necessary information on focus, process and outcomes is present and presented in a coherent way	
report relevance	standard not reached	the understanding of the focus, process and outcomes of the investigation is obscured by the presence of inappropriate or irrelevant information		relevant and concise thereby facilitating a ready understanding of the focus, process and outcomes of the investigation	
terminology	standard not reached	there are many errors in the use of subject specific terminology and conventions		the use of subject specific terminology and conventions are appropriate and correct; any errors do not hamper under-standing	