Topic 6 Fields at work

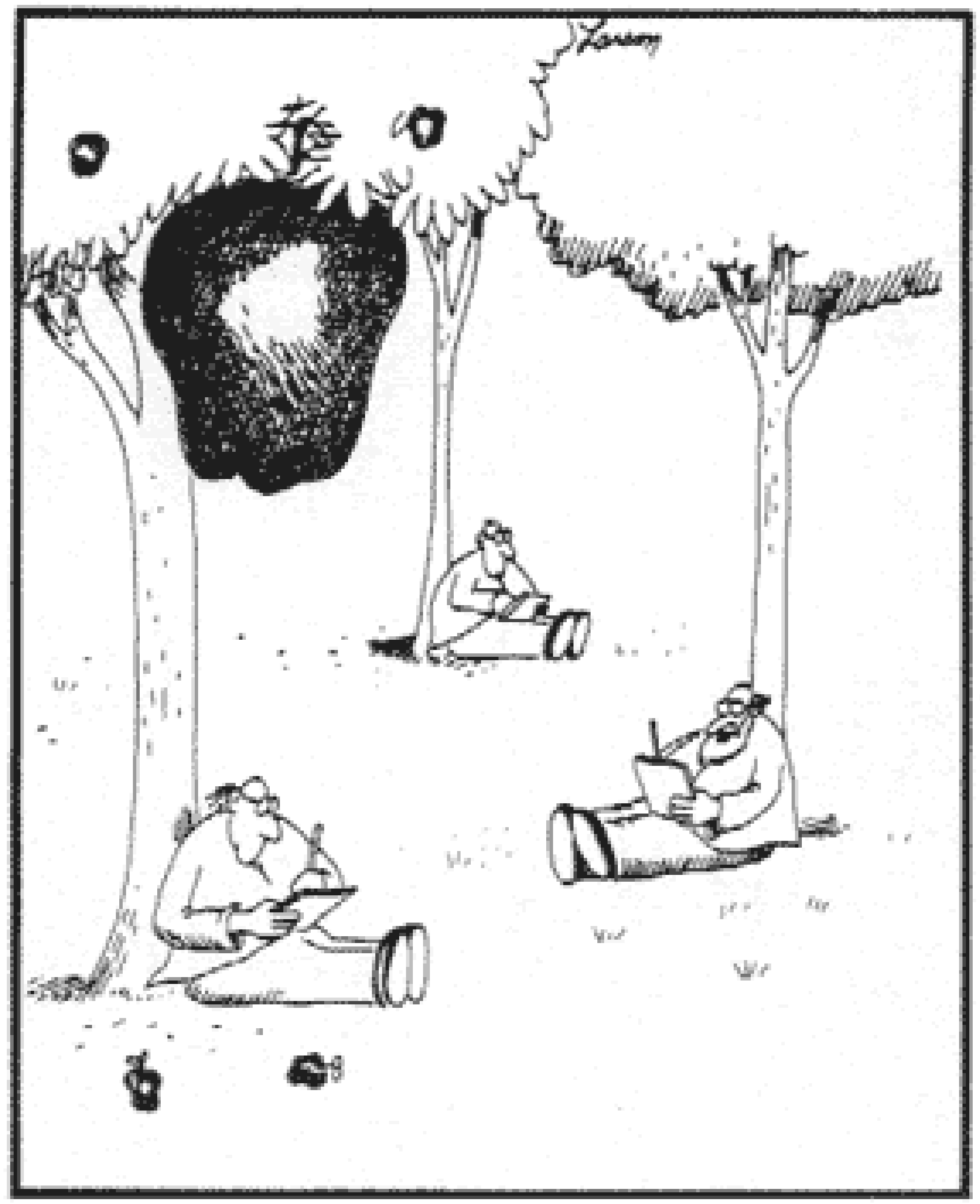

CAUTION: Gravity at work

Gravity

Just a theory.

Newton's Gravitation

- Newton's most original contribution!
- All objects in the universe exert a gravitational pull on each other
- F depends on:
  - d
  - m



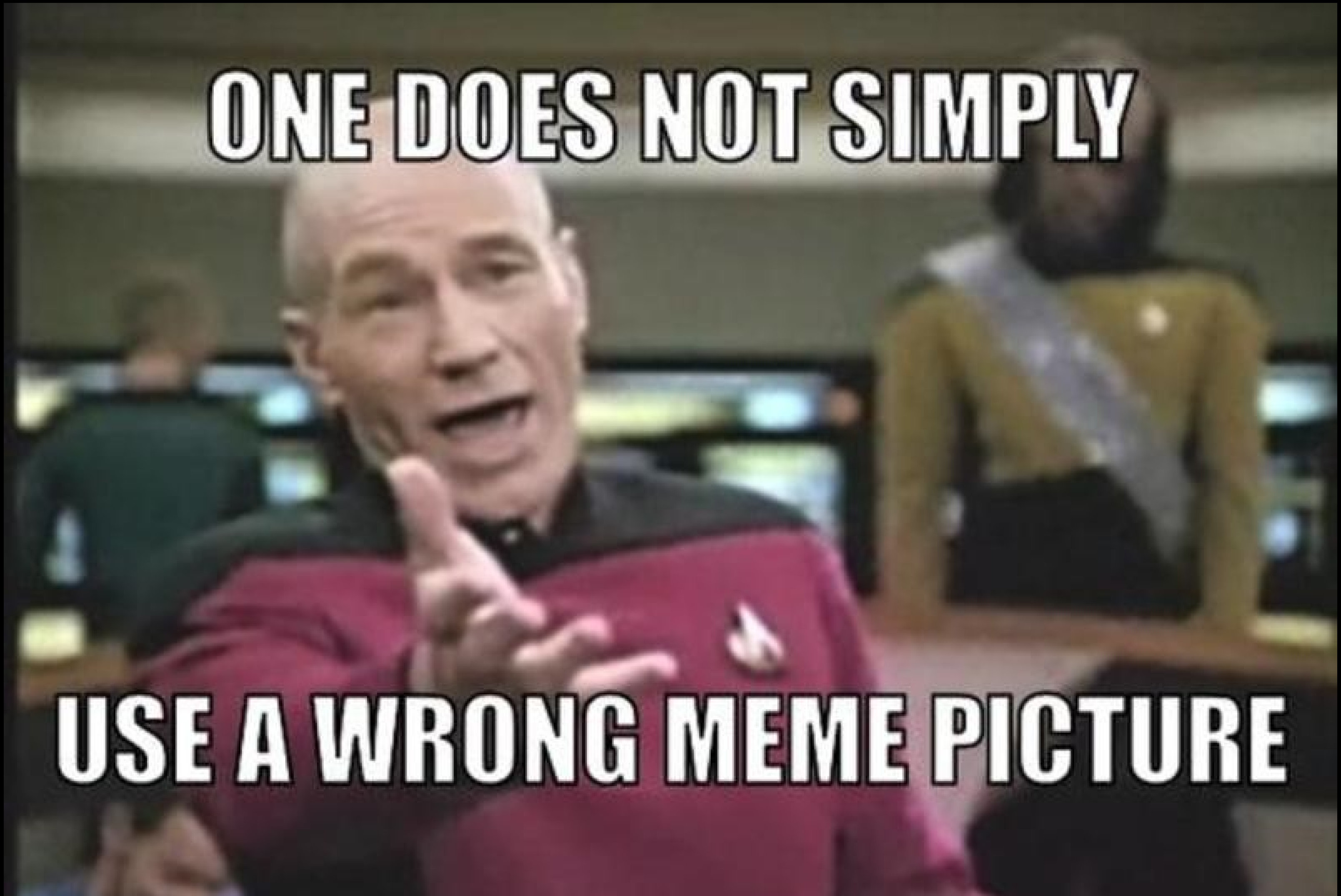
"Nothing yet . . . How about you, Newton?"

Gravitational Definitions

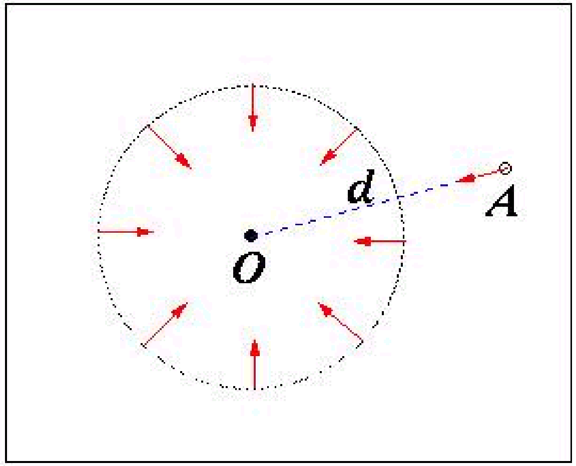
- This is where most people mix up problems
- Mass, Force or Field?
  - 25N
  - Weight
  - 5kg
  - 9.8N/kg
  - Apparent Weight
  - Gravitational acceleration
  - 25lb!? 2
  - 1.63m/s^2

One does not simply…

…mix up mass and weight



Universal gravitation:



$$F_g = \frac{GMm}{r^2}$$

- Where G is the universal gravitational constant

The problem in determining "G" is in measuring the feeble force between two ordinary (and "weighable") objects

Ex 1: Find the force of attraction between Rachel (50 kg) and Matt(80 kg) if they sit 2 m apart.

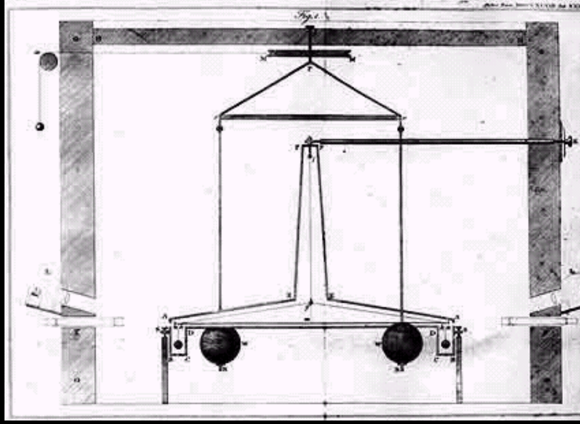
$$F_g = \frac{GMm}{r^2}$$

$$F_g = \frac{6.67\times10^{-11}\ Nm^2/kg^2 \cdot 80kg \cdot 50kg}{(2m)^2}$$

$$F_g = 6.7\times10^{-8}\,N$$

Too small to measure directly: (

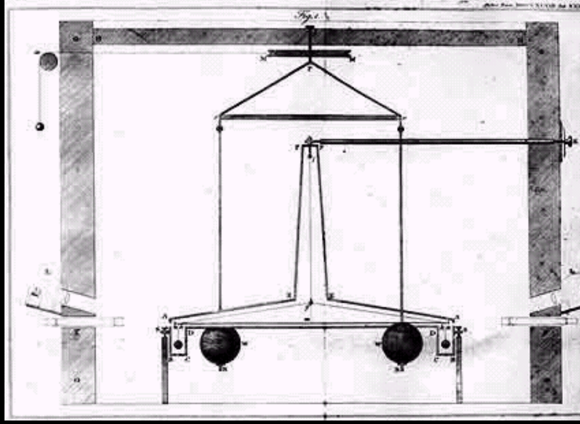
Cavendish's "Weigh the Earth" Experiment



- If we can get a measurable force for two known masses, we can find G
- Once G is known, we can "weigh" anything!
- Find G if Cavendish calculated the force between the 158 kg mass and the 0.73kg mass (when their centers are 25cm apart) to be:

$$F_g = 1.24 \times 10^{-7} N$$

$$F_g = \frac{GMm}{r^2}$$



$$G = \frac{F_g r^2}{Mm} = \frac{1.24\times 10^{-7}\,N(0.25m)^2}{158kg\cdot 0.73kg}$$

$$G = 6.7\times 10^{-11}\,Nm^2/kg^2$$

Ex 2: find mass

- Use the gravitational constant to find the Earth's mass

$$F = \frac{GMm}{r^2}$$

$$M = \frac{Fr^2}{Gm}$$

$$M = \frac{9.81N \cdot \left(6.38 \times 10^6 m\right)^2}{6.67 \times 10^{-11} \frac{Nm^2}{kg^2} \cdot 1.00kg}$$

- Ex 2: find the gravitational field strength "g" at an *altitude* of 130 km. Careful!

$$F_g = mg$$

$$g = \frac{GM}{r^2}$$

$$g = \frac{6.67\times10^{-11}\, Nm^2/kg^2 \cdot 5.98\times10^{24}\, kg}{(6.38\times10^6\, m + 1.3\times10^5\, m)^2}$$

$$g = 9.41\, N/kg$$

Ex 4: find mass

- Use the Earth's orbit to find the mass of the Sun

$$F = ma$$

$$\frac{GMm}{r^2} = \frac{m4\pi^2 r}{T^2}$$

$$M = \frac{v^2 r}{G}$$

$$M = \frac{\left(3.0\times10^4 \, {}^{m}\!/_{s}\right)^2 1.5\times10^{11}\, m}{6.67\times10^{-11}\, {}^{Nm^2}\!/_{kg^2}}$$

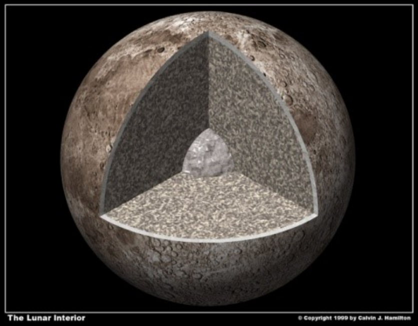
$$M = 2.0\times10^{30}\, kg$$

Exercises

- P. 183-4 #1-3

Ex. 3: find g on the moon.

$$g = \frac{GM}{r^2}$$



$$g = \frac{6.67\times10^{-11}\,Nm^2/kg^2 \cdot 7.35\times10^{22}\,kg}{(1.74\times10^{6}\,m)^2}$$

$$g = 1.62\,N/kg$$

Ex 5: find speed

- How fast would you need to orbit a 10 Solar mass black hole at a distance of 100km?

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$v = \sqrt{\frac{GM}{r}}$$

$$v = \sqrt{\frac{6.67\times10^{-11}\, Nm^2/kg^2 \left(1.98\times10^{31}\, kg\right)}{1.00\times10^{5}\, m}}$$

$$v = 1.15\times10^{8}\, m/s$$

Energy

Gravitational Potential Energy

- Switching from relative potential energy $mg\Delta h$
- Absolute Grav $E_p=-GMm/r$
- This is negative since gravity is always an attractive force. We have to put energy into the system to separate the masses.
- To find the energy needed to lift a mass off the surface of a planet, simply subtract the potential energy before and after

Ex 1: How much potential energy?

- What is the gravitational energy of a 55kg "little prince" standing on the surface of a 100m wide, $3x10^9$kg asteroid?

$$E_p = \frac{-GMm}{r}$$

$$E_p = \frac{-6.67\times10^{-11}N\,m^2/kg^2\,(3.0\times10^9kg)55kg}{50m}$$

$$E_p = -0.22J$$

Ex 2: ΔE

- How much energy is required to lift off a 2900 Ton Saturn V rocket 100 km straight off the surface of the planet?

$$\Delta E_p = \Delta\left(\frac{-GMm}{r}\right)$$

$$\Delta E_p = -GMm\left(\frac{1}{\left(6.48\times10^6\right)} - \frac{1}{\left(6.38\times10^6\right)}\right)$$

$$\Delta E_p = -GMm\left(\frac{1}{r} - \frac{1}{r_0}\right)$$

$$\Delta E_p = 2.8TJ$$

Ex 4: Escape velocity

- Find the escape velocity for:
  - the Earth
  - the Moon
  - a 10 km diameter comet
- This is the minimum velocity for which kinetic energy just balances potential, or:

$$E_T = E_k + E_p = 0$$

$$E_k = -E_p$$

$$\frac{1}{2} m v^2 = \frac{GMm}{r}$$

$$v = \sqrt{\frac{2GM}{r}}$$

- The Earth:

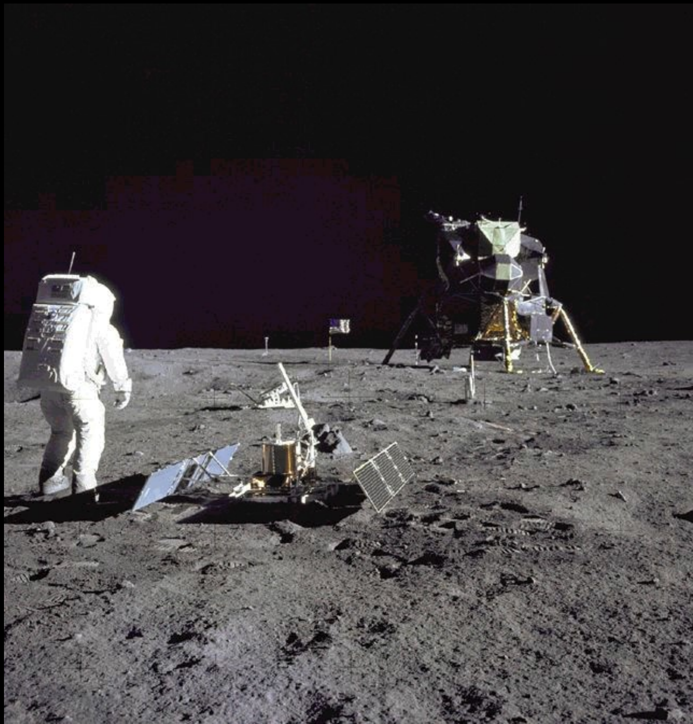
$$v = \sqrt{\frac{2GM}{r}}$$

$$v = 11.1 \, {}^{km}\!/_{s}$$

- The Moon:

$$v = \sqrt{\frac{2GM}{r}}$$

$$v = 2.4 \, {}^{km}\!/_{s}$$



- The comet: mass?

$$M = \rho V = \rho \frac{4}{3}\pi r^3$$

$$M = 1000 \, {}^{kg}\!/\!{}_{m^3} \, \frac{4}{3}\pi (5000m)^3$$

$$M = 5.24 \times 10^{14} kg$$

$$v = \sqrt{\frac{2GM}{r}}$$

$$v = \sqrt{\frac{2 \cdot G \cdot 5.24 \times 10^{14} kg}{5000m}}$$

$$v = 3.7 \, {}^{m}\!/\!{}_{s}$$

- The comet continued: could you jump clear off the surface?

$$v_{esc} = 3.7\, {}^{m}/_{s}$$

$$g = \frac{GM}{r^2}$$

$$g = \frac{G \cdot 5.24 \times 10^{14}\, kg}{(5000m)^2}$$

$$g = 0.0014 m/s^2$$

$$F_{net} = ma$$

$$a = \frac{F_g + F_N}{m} = \frac{mg_c - 2mg_E}{m}$$

$$a = g_c - 2g_E$$

$$a = 19.6\, {}^{m}/_{s^2}$$

$$v^2 = v_0^{\ 2} + 2ad$$

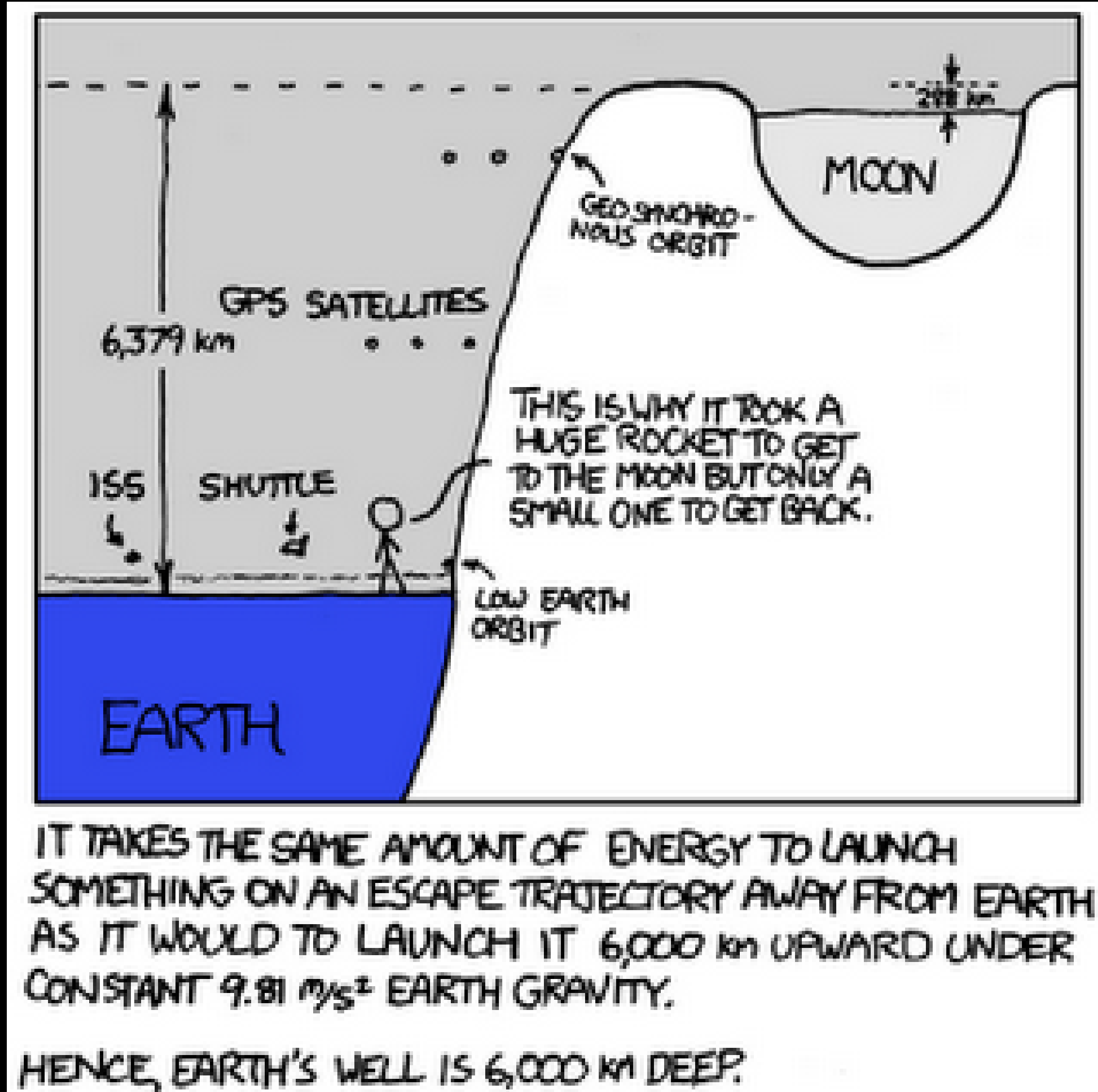
$$v = \sqrt{2ad} = \sqrt{2 \cdot 19.6 \, {}^{m}\!/_{s^2} \cdot 0.5m}$$

$$v = 4.4\, m/s$$

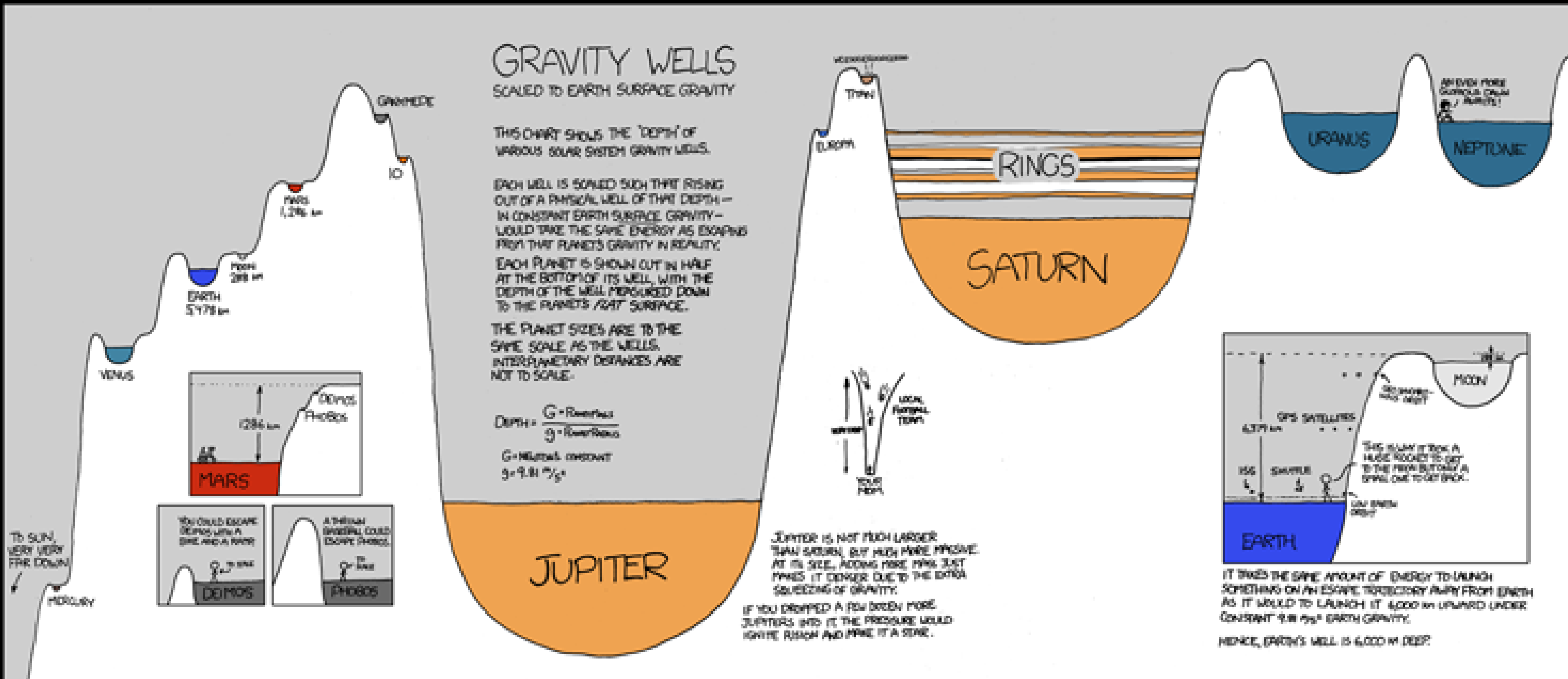
- The lesson? Be careful when jumping around on a comet!

Exercises

- P. 127 #1-3



6,379 km
GPS SATELLITES
GEOSYNCHRONOUS ORBIT
MOON
THIS IS WHY IT TOOK A HUGE ROCKET TO GET TO THE MOON BUT ONLY A SMALL ONE TO GET BACK.
ISS
SHUTTLE
LOW EARTH ORBIT
EARTH
IT TAKES THE SAME AMOUNT OF ENERGY TO LAUNCH SOMETHING ON AN ESCAPE TRAJECTORY AWAY FROM EARTH AS IT WOULD TO LAUNCH IT 6,000 km UPWARD UNDER CONSTANT 9.81 m/s² EARTH GRAVITY.
HENCE, EARTH'S WELL IS 6,000 km DEEP.



GRAVITY WELLS
SCALED TO EARTH SURFACE GRAVITY
THIS CHART SHOWS THE 'DEPTH' OF VARIOUS SOLAR SYSTEM GRAVITY WELLS.
EACH WELL IS SCALED SUCH THAT RISING OUT OF A PHYSICAL WELL OF THAT DEPTH — IN CONSTANT EARTH SURFACE GRAVITY — WOULD TAKE THE SAME ENERGY AS ESCAPING FROM THAT PLANET'S GRAVITY IN REALITY.
EACH PLANET IS SHOWN CUT IN HALF AT THE BOTTOM OF ITS WELL, WITH THE DEPTH OF THE WELL MEASURED DOWN TO THE PLANET'S FLAT SURFACE.
THE PLANET SIZES ARE TO THE SAME SCALE AS THE WELLS. INTERPLANETARY DISTANCES ARE NOT TO SCALE.
G = NEWTON'S CONSTANT
g = 9.81 m/s²
TO SUN, VERY VERY FAR DOWN
MERCURY
VENUS
EARTH
MOON
MARS
GANYMEDE
IO
JUPITER
EUROPA
TITAN
RINGS
SATURN
URANUS
NEPTUNE
MARS
DEIMOS
PHOBOS
DEIMOS
PHOBOS
JUPITER IS NOT MUCH LARGER THAN SATURN, BUT MUCH MORE MASSIVE. AT ITS SIZE, ADDING MORE MASS JUST MAKES IT DENSER DUE TO THE EXTRA SQUEEZING OF GRAVITY.
IF YOU DROPPED A FEW DOZEN MORE JUPITERS INTO IT, THE PRESSURE WOULD IGNITE FUSION AND MAKE IT A STAR.
MOON
GPS SATELLITES
ISS
SHUTTLE
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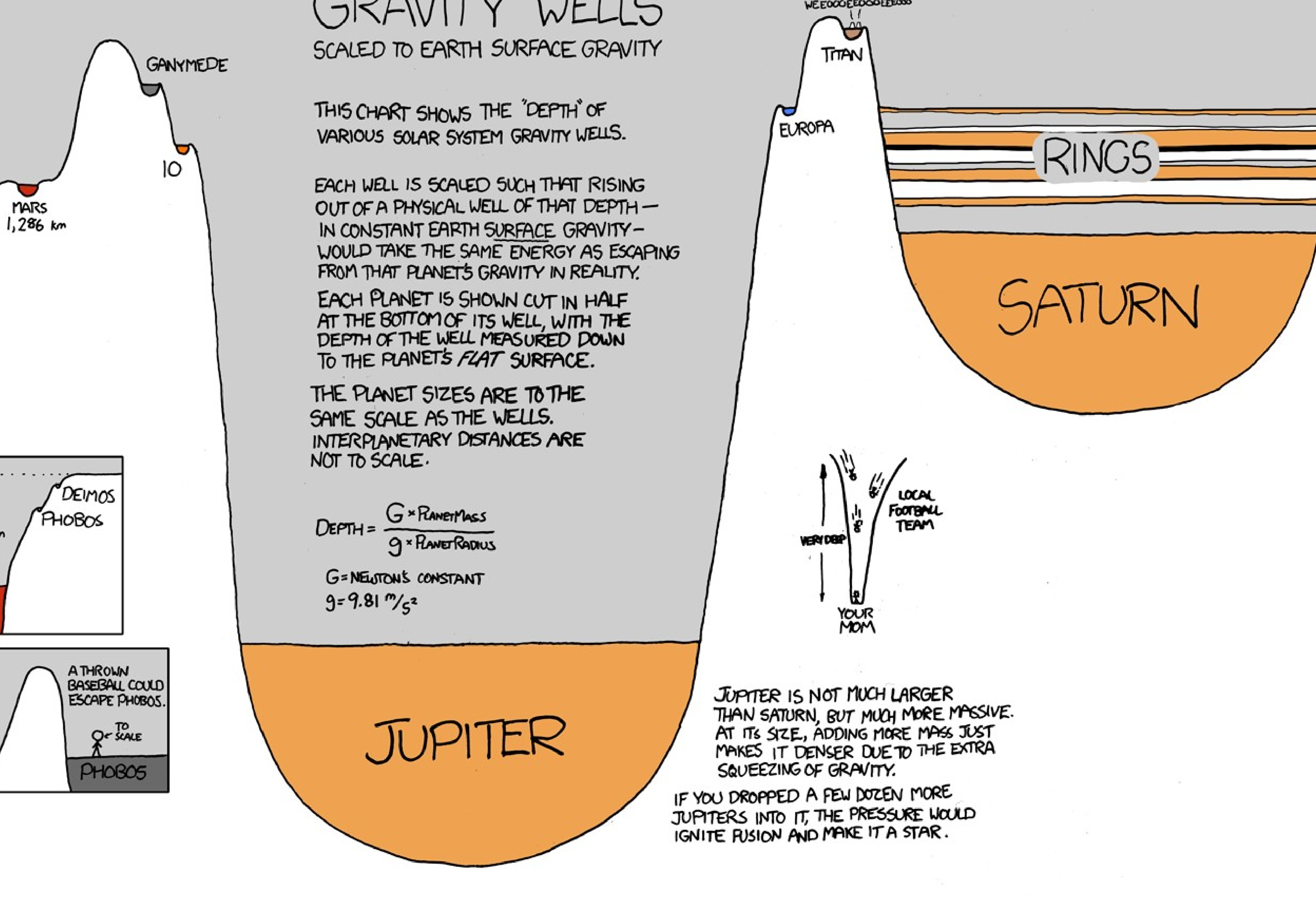
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$\text{DEPTH} = \dfrac{G \times \text{PLANET MASS}}{g \times \text{PLANET RADIUS}}$
G = NEWTON'S CONSTANT
$g = 9.81\ m/s^2$
GANYMEDE
IO
MARS
1,286 km
DEIMOS
PHOBOS
A THROWN BASEBALL COULD ESCAPE PHOBOS.
TO SCALE
PHOBOS
JUPITER
TITAN
EUROPA
RINGS
SATURN
VERY DEEP
LOCAL FOOTBALL TEAM
YOUR MOM
JUPITER IS NOT MUCH LARGER THAN SATURN, BUT MUCH MORE MASSIVE. AT ITS SIZE, ADDING MORE MASS JUST MAKES IT DENSER DUE TO THE EXTRA SQUEEZING OF GRAVITY.
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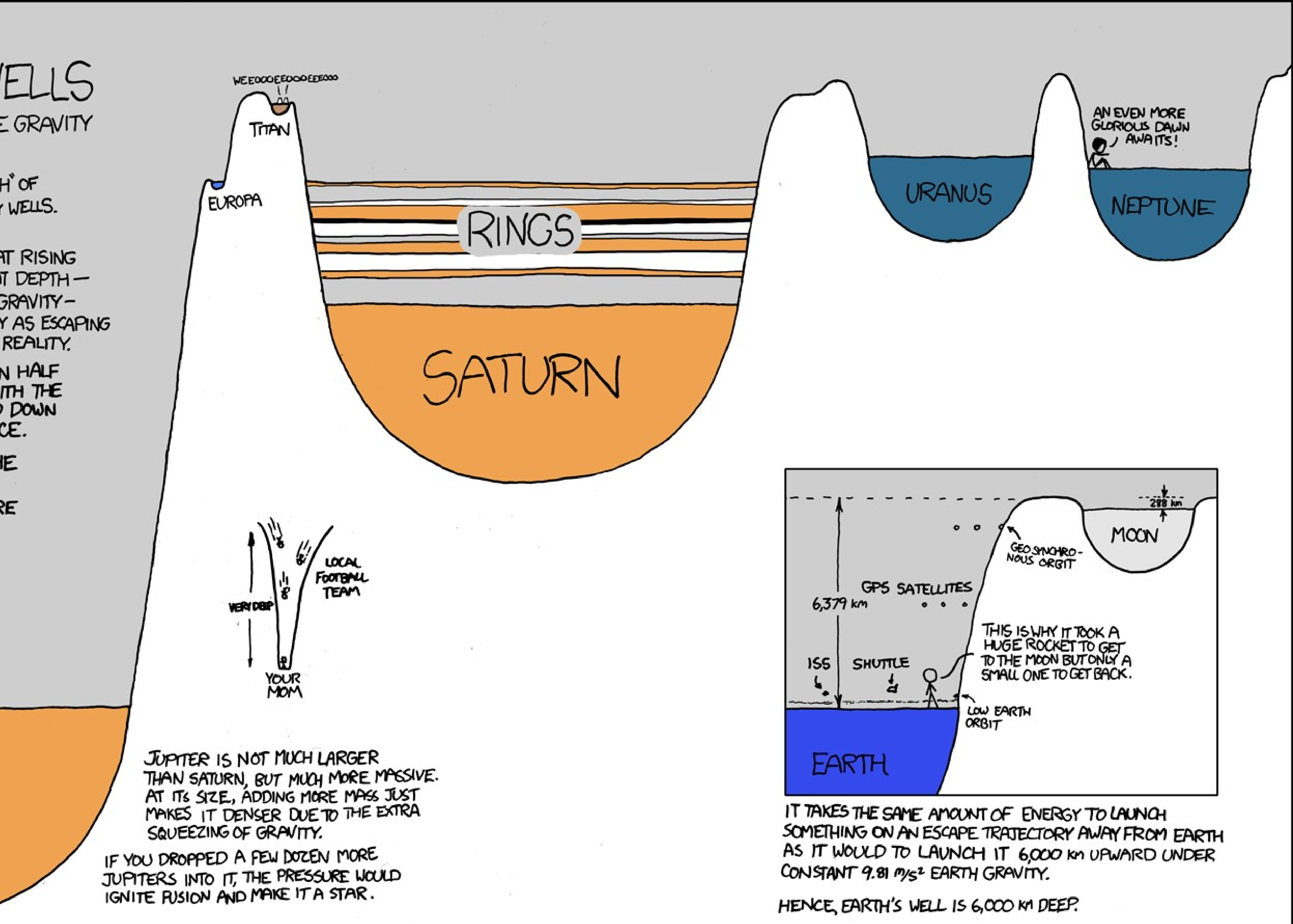

VELLS
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TITAN
EUROPA
RINGS
SATURN
URANUS
AN EVEN MORE GLORIOUS DAWN AWAITS!
NEPTUNE
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288 km
MOON
GEOSYNCHRONOUS ORBIT
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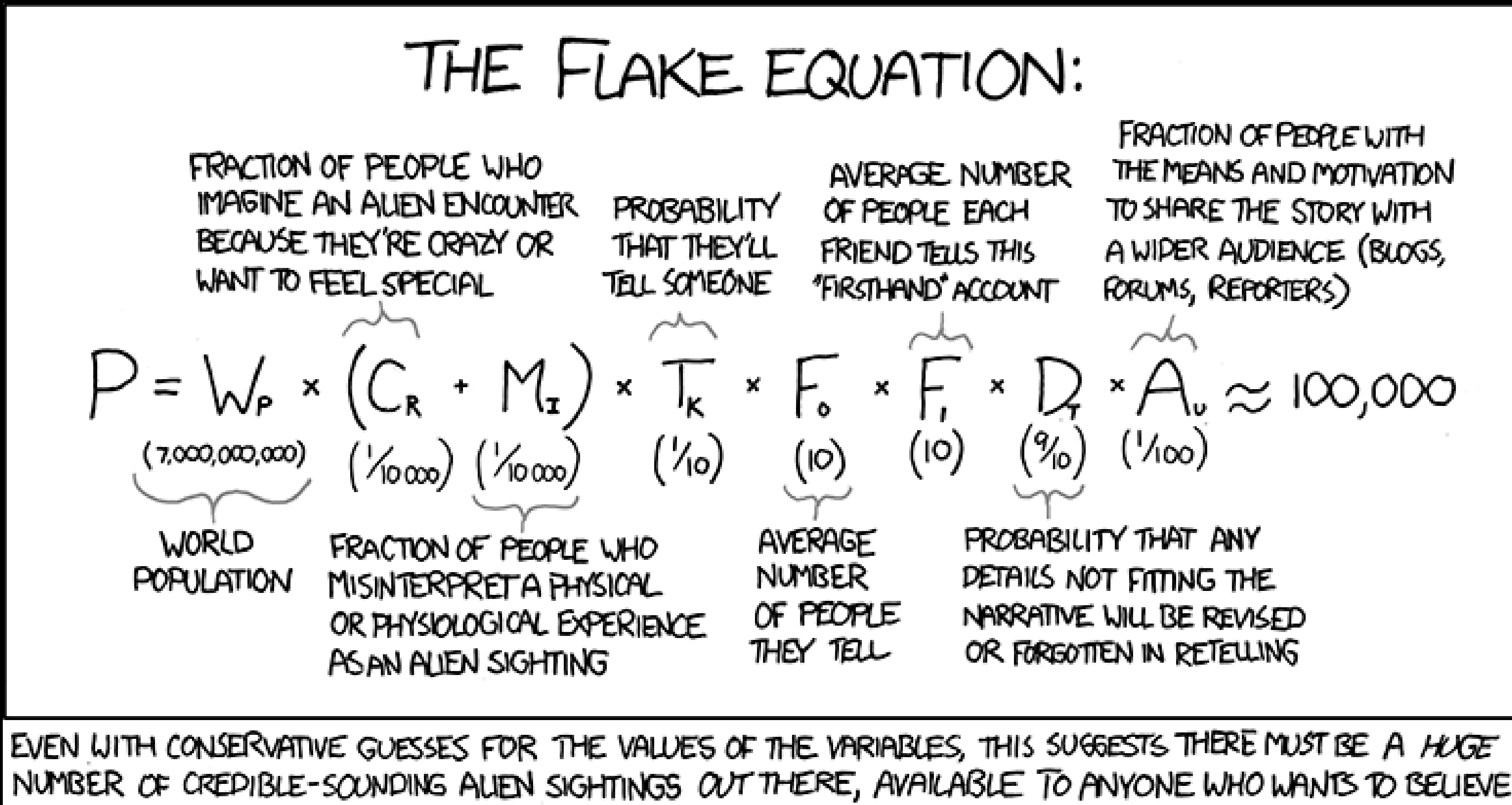
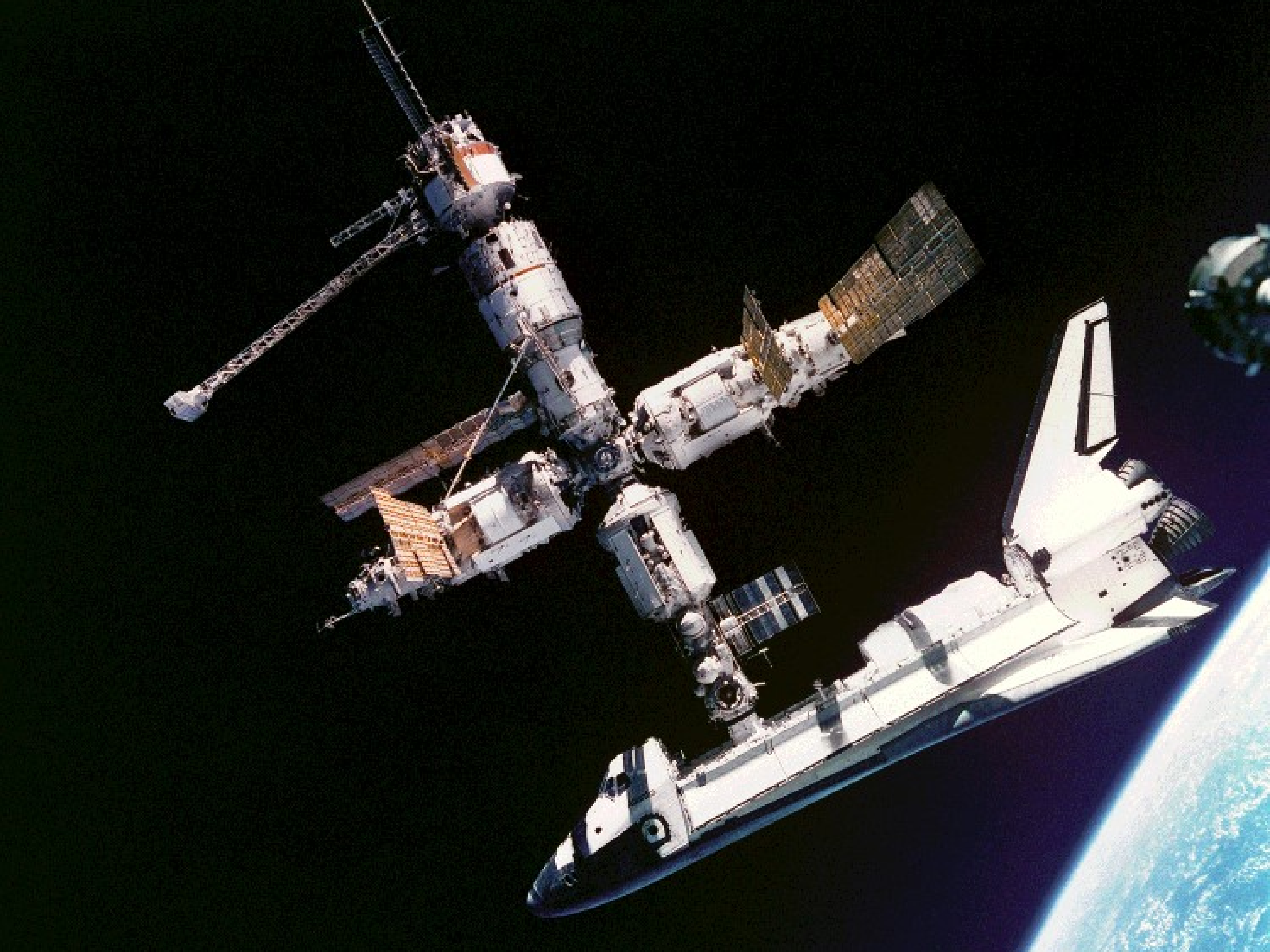

THE FLAKE EQUATION:
FRACTION OF PEOPLE WHO IMAGINE AN ALIEN ENCOUNTER BECAUSE THEY'RE CRAZY OR WANT TO FEEL SPECIAL
PROBABILITY THAT THEY'LL TELL SOMEONE
AVERAGE NUMBER OF PEOPLE EACH FRIEND TELLS THIS "FIRSTHAND" ACCOUNT
FRACTION OF PEOPLE WITH THE MEANS AND MOTIVATION TO SHARE THE STORY WITH A WIDER AUDIENCE (BLOGS, FORUMS, REPORTERS)
$P = W_P \times (C_R + M_I) \times T_K \times F_O \times F_I \times D_T \times A_U \approx 100{,}000$
(7,000,000,000) (1/10 000) (1/10 000) (1/10) (10) (10) (9/10) (1/100)
WORLD POPULATION
FRACTION OF PEOPLE WHO MISINTERPRET A PHYSICAL OR PHYSIOLOGICAL EXPERIENCE AS AN ALIEN SIGHTING
AVERAGE NUMBER OF PEOPLE THEY TELL
PROBABILITY THAT ANY DETAILS NOT FITTING THE NARRATIVE WILL BE REVISED OR FORGOTTEN IN RETELLING
EVEN WITH CONSERVATIVE GUESSES FOR THE VALUES OF THE VARIABLES, THIS SUGGESTS THERE MUST BE A HUGE NUMBER OF CREDIBLE-SOUNDING ALIEN SIGHTINGS OUT THERE, AVAILABLE TO ANYONE WHO WANTS TO BELIEVE!

Orbital Velocity

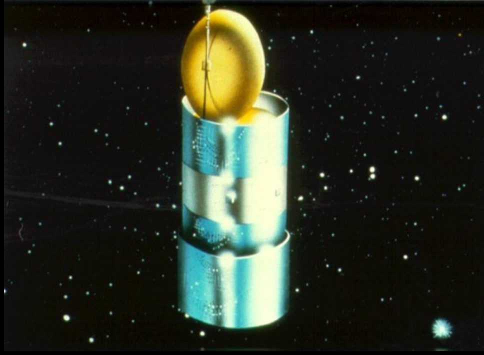
Orbital velocity

- This can be thought of as the half way point towards total freedom: orbital energy is half the potential energy at that point.
- This gives us:

$$v = \sqrt{\frac{GM}{r}}$$



Ex 3: Geosynchronous Orbit



- What period should a satellite in geosynchronous orbit have?

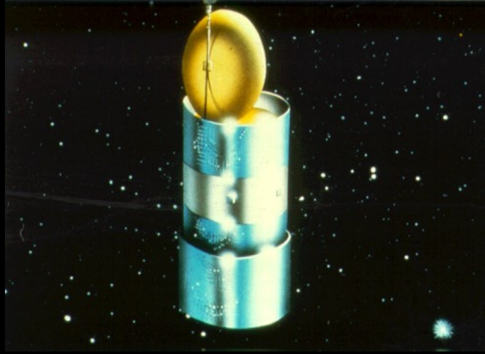
- How fast is a satellite in geosynchronous orbit (42000 km)?

$$v = \sqrt{\frac{GM}{r}}$$

$$v = \sqrt{\frac{6.67\times10^{-11}\cdot5.98\times10^{24}}{4.2\times10^{7}}}$$

$$v = 3.1 km\cdot s^{-1}$$

Gravitational Potential



- This is energy per unit mass

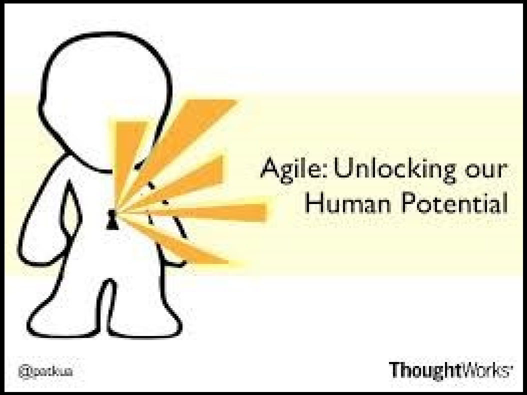
$$V = -\frac{GM}{r}$$

- Ex: what is the potential at a point 3500 km above the surface of the Earth?

$$V = -\frac{6.67\times10^{-11}\cdot5.98\times10^{24}}{6.38\times10^{6}+3.5\times10^{6}}$$

$$V = -4.0\times10^{7}\,J\cdot kg^{-1}$$

What is your maximum potential?

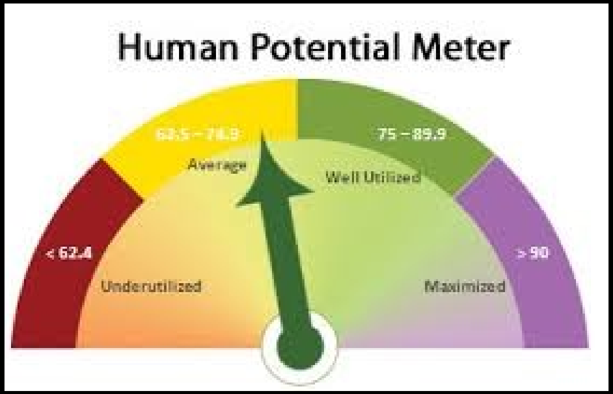


$$V = -\frac{GM}{r}$$

$$V = -\frac{6.67\times10^{-11}(80)}{0.2}$$

$$V = -30nJ \cdot kg^{-1}$$

Can you graph your potential?



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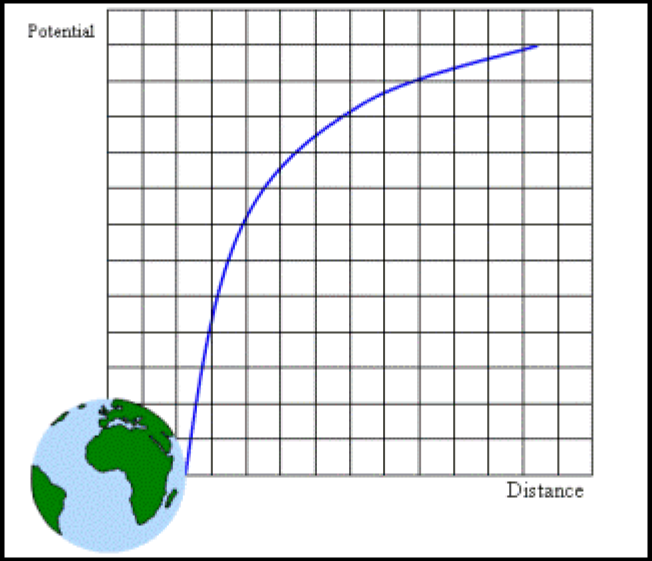


Gravitational Potential

- This can also be expressed as V=gh. Ex: what is the gravitational potential on NorKam's roof, relative to the ground?

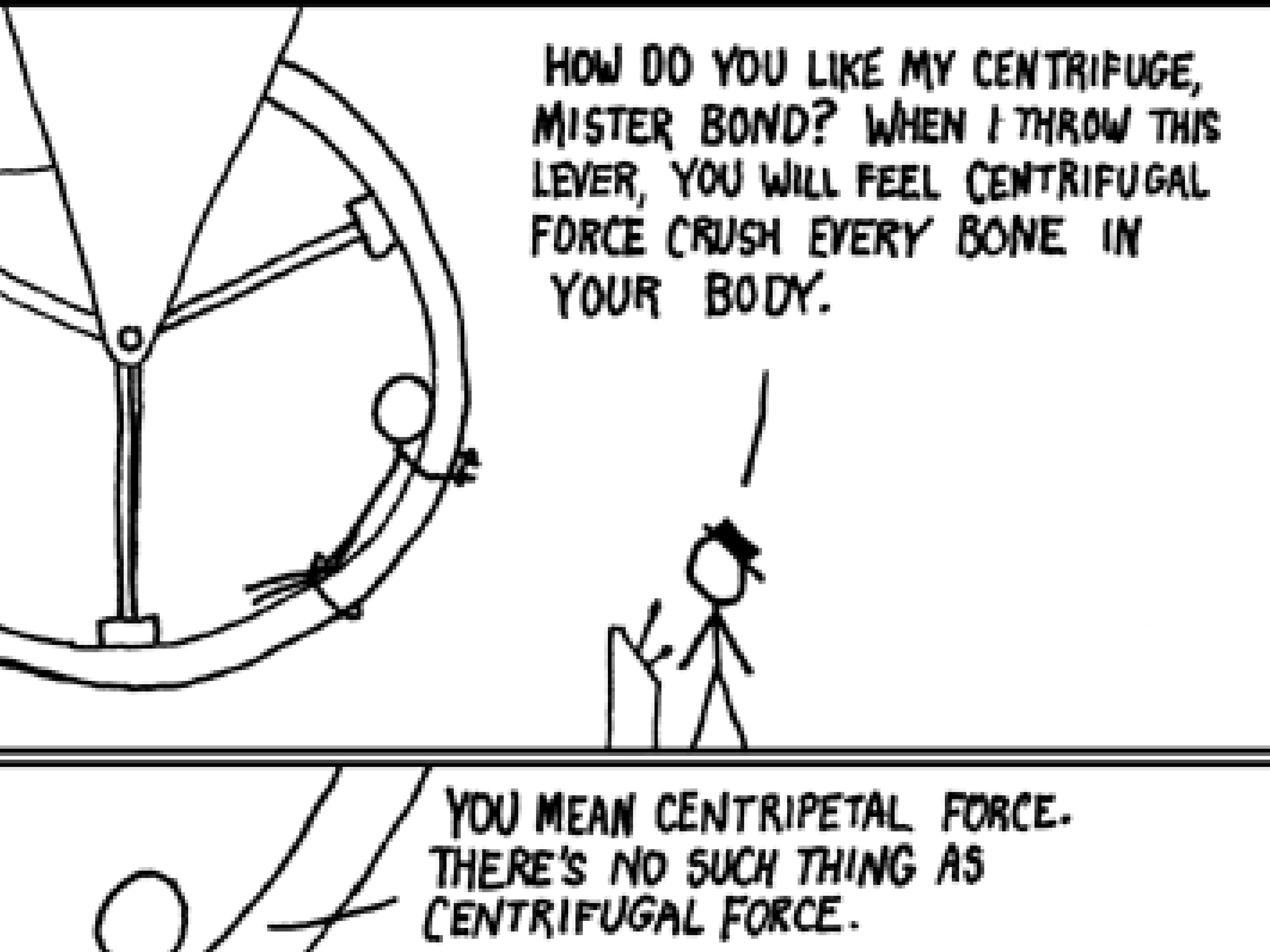
$$V = gh = 5 \cdot 9.8 = 49 J \cdot kg^{-1}$$

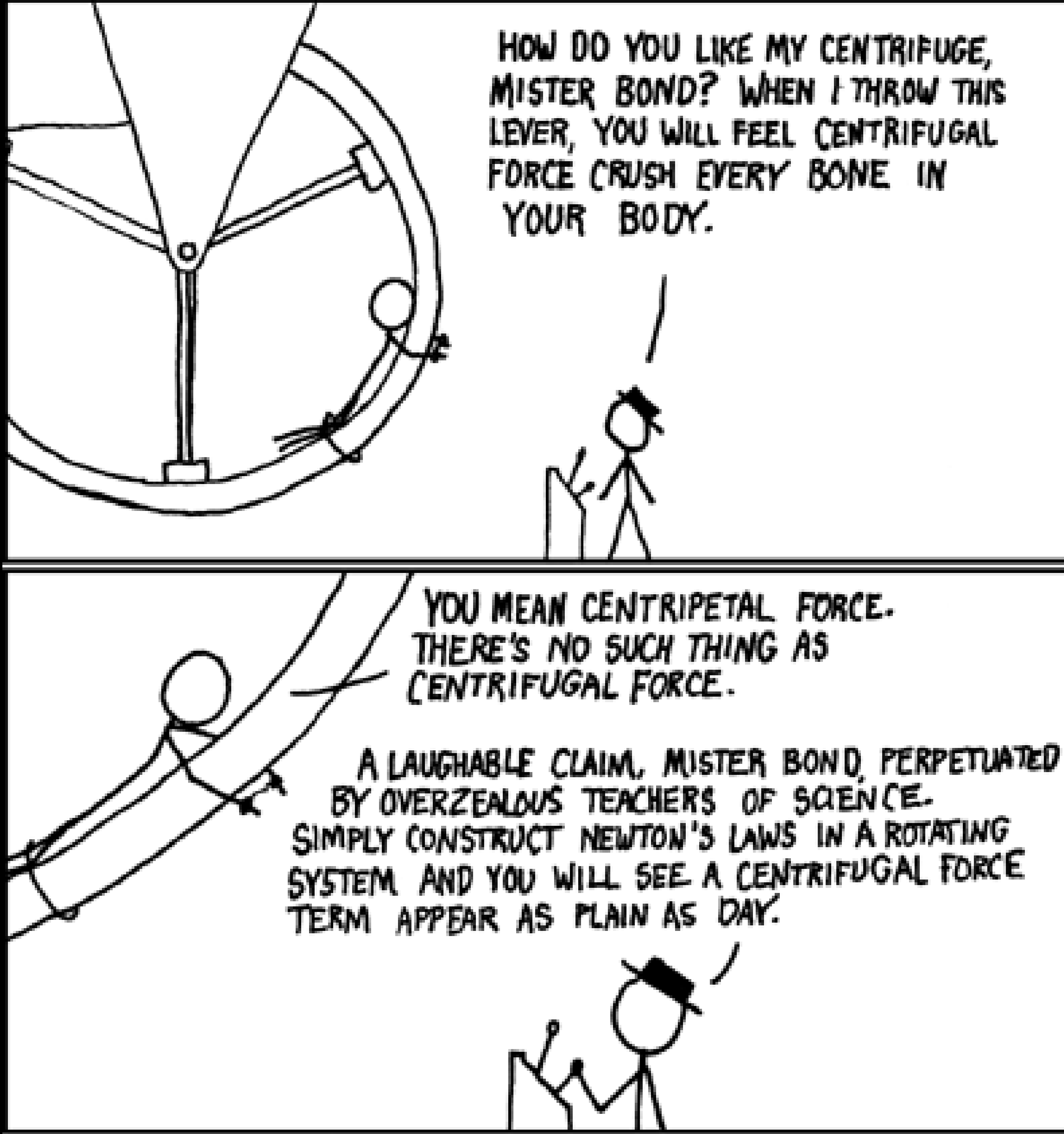
- Ex: what is the potential difference on the Crystal chair?

$$\Delta V = g\Delta h = 244 \cdot 9.8 = 2400 J \cdot kg^{-1}$$

Exercises

- Finish up to #5 p. 185
- Continue up to #11 p. 190
- Keep working on IA


HOW DO YOU LIKE MY CENTRIFUGE, MISTER BOND? WHEN I THROW THIS LEVER, YOU WILL FEEL CENTRIFUGAL FORCE CRUSH EVERY BONE IN YOUR BODY.
YOU MEAN CENTRIPETAL FORCE. THERE'S NO SUCH THING AS CENTRIFUGAL FORCE.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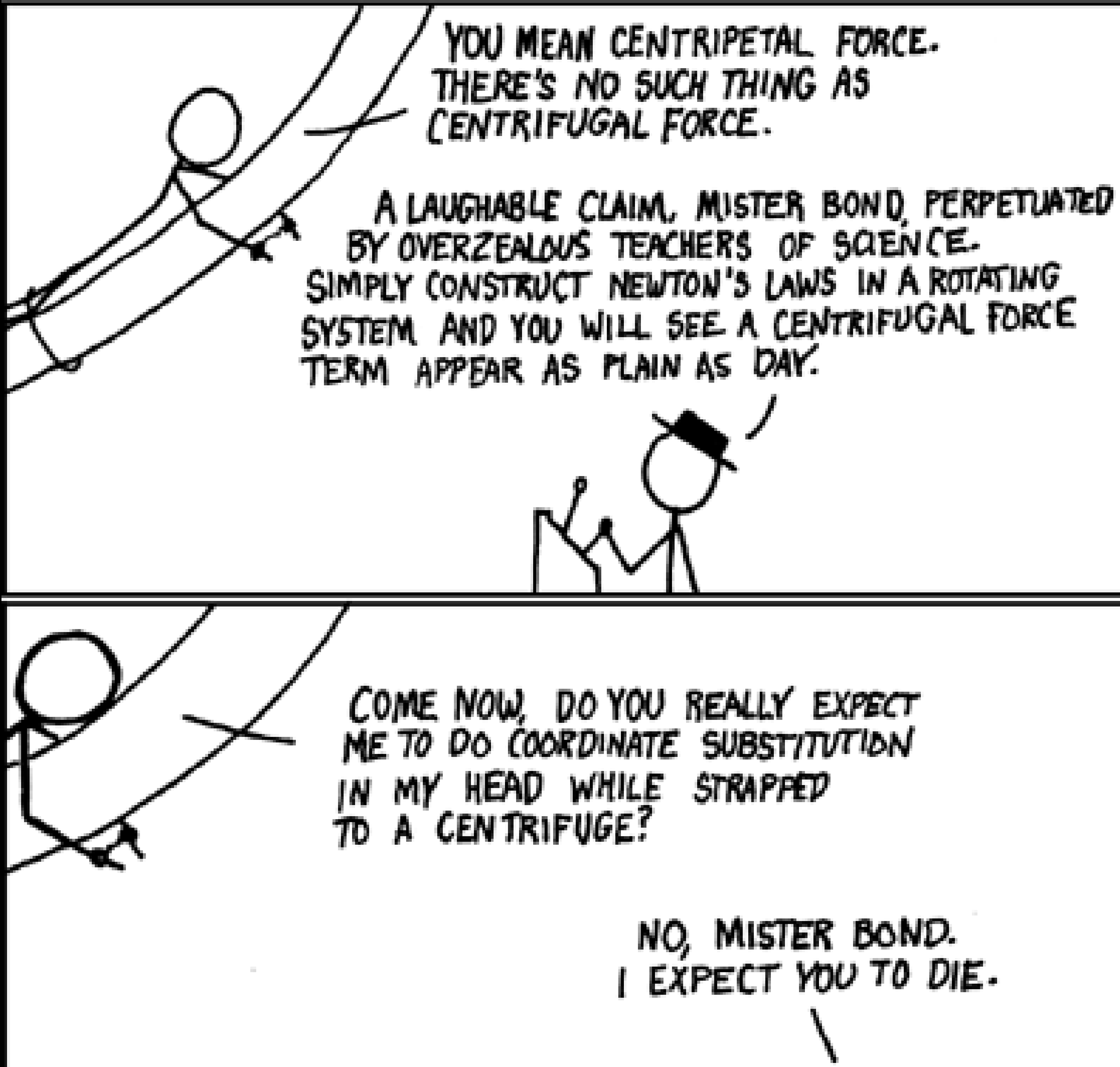
A LAUGHABLE CLAIM, MISTER BOND, PERPETUATED BY OVERZEALOUS TEACHERS OF SCIENCE. SIMPLY CONSTRUCT NEWTON'S LAWS IN A ROTATING SYSTEM AND YOU WILL SEE A CENTRIFUGAL FORCE TERM APPEAR AS PLAIN AS DAY.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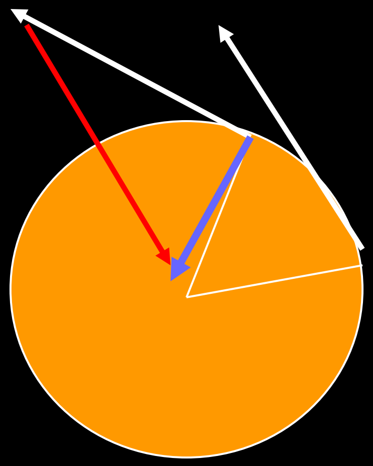
COME NOW, DO YOU REALLY EXPECT ME TO DO COORDINATE SUBSTITUTION IN MY HEAD WHILE STRAPPED TO A CENTRIFUGE?
NO, MISTER BOND. I EXPECT YOU TO DIE.

Centripetal Acceleration

- We can have acceleration with constant speed
- If a=Δv/ Δ t we find the direction of a is towards the center of the circle

$$\Delta v = v - u$$ $$= v + \left(-u\right)$$



Magnitude of a_c

$$a_c = \frac{v^2}{r}$$

- We can also express this in terms of period of the rotation since v=d/t
- $v=d/t=2\pi r/T$
- So we also have

$$a_c = \frac{4\pi^2 r}{T^2}$$

Ex 1: find the acceleration

- A 2kg mass is swung at the end of a 0.5 m rope with a period of rotation of 0.75s. Find a

$$a_c = \frac{4\pi^2 r}{T^2}$$

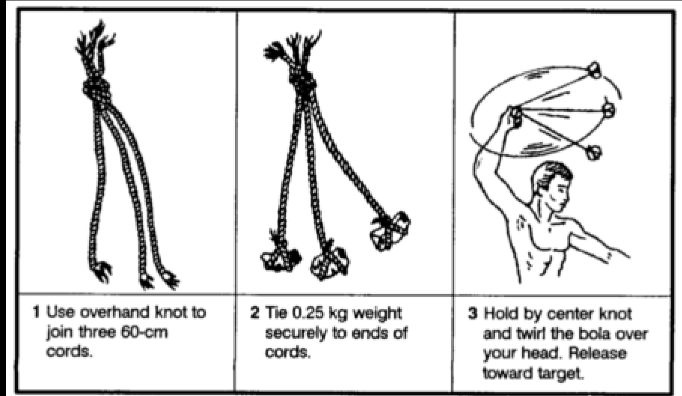


Figure 12-7. Bola.

$$a_c = \frac{4\pi^2 \cdot 0.5m}{(0.75s)^2}$$

$$a_c = 35 \, {}^{m}\!/_{s^2}$$

Ex 2: find force

- The Earth travels in its orbit at a speed of about 30 km/s. What force is necessary to keep it in its orbit?

$$a_c = \frac{v^2}{r}$$

$$F = ma_c = \frac{mv^2}{r}$$

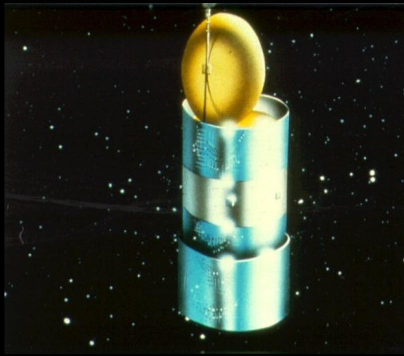
$$F = \frac{5.98\times10^{24}\,kg\left(30000\,{}^{m}\!/\!{}_{s}\right)^2}{1.5\times10^{11}\,m}$$

$$F = 3.6\times10^{22}\,N$$

Try it!

- Lab 4-1 p. 110 Physics Two

Ex 3: Geosynchronous Orbit



- What orbital radius should this satellite have?

$$a_c = g$$

$$\frac{4\pi^2 r}{T^2} = \frac{GM}{r^2}$$

$$4\pi^2 r^3 = GMT^2$$

$$r = \sqrt[3]{\frac{GMT^2}{4\pi^2}}$$

$$r = \sqrt[3]{\frac{\left(6.67\times10^{-11}\right)\left(5.98\times10^{24}\right)\left(86164\right)^2}{4\pi^2}}$$

$$r = 4.22\times10^{7}\,m$$

Exercises

- Continue up to #19 p. 194
- Keep working on IA